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ABSTRACT

This monograph provides activities for beginning algebra students to develop an understanding of the meaning and use of letter symbols, to show applications of algebra in their own lives, and to develop an understanding of mathematical processes through investigation. The activities included were developed by considering the students' mathematical knowledge, interests, maturity level, and everyday living. Several attributes of a student-centered classroom are listed. The lessons fall under four headings: (1) "Bridges from Arithmetic to Algebra"; (2) "Estimation & Conjecture"; (3) "Situational Lessons"; and (4) "Investigations." Learning materials and answers to questions in the lessons are appended. (YP)

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Motivators for Algebra One

A SITUATIONAL APPROACH

Lesson Plans for the New California Framework and the New National Standards

Louis McCreery
Office of Continuing Education
California State Polytechnic University, Pomona
1989

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THE MATHEMATICS FRAMEWORK FOR CALIFORNIA PUBLIC SCHOOLS THE CURRICULUM AND EVALUATION STANDARDS FOR SCHOOL MATHEMATICS, AND THIS SYLLABUS

The Framework and the Standards have similar philosophies. The approach in this monograph follows closely in the same vein and can be used with either work.

For instance, each emphasizes that true understanding takes place only when new concepts are associated with ideas familiar to the student. In the language of the syllabus: "Start where the students are".

Each calls for problems that come from the "real world" or "situations" again from the student's life.

Questioning and conjecture, estimation and discovery are emphasized, particularly in the Standards. Motivators provides several investigations with experiences in these elements as well as mathematical reasoning.

Recent brain research indicates that an involved learner's electrical impulses are stronger, the energy level in the synapses increases when the student is actively learning, much more than when he or she is going thru rote processes. We knew that involved students learned better, now we have a scientific basis for that knowledge. All three works emphasize student involvement.

The students become involved when they have a personal stake in the answer thru estimation and conjecture, when the answer on a problem is related directly to their life right now, when they are challenged, when they have a sense of growth, when they discover, when the material builds on what they know, when it is satisfying, and when it is fun.

The author feels that the ideas expressed here are very much in line with the future of mathematics education.

INTRODUCTION

The lessons in this book are student-centered. It is the students' world, their interests and their abilities with which we deal. It is our desire to challenge them, raise questions that they want answered, to check out their estimates and conjectures, to give them the opportunity to investigate and discover. They will find the value and excitement of mathematics for themselves and the world around them.

To quote the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989), they will become "mathematically literate workers with the ability to explore, to conjecture, and to reason logically, as well as use a variety of mathematical methods to effectively solve problems."

These lessons start where the students are--where they are in mathematical knowledge, where they are in interests, where they are in maturity, where they are in everyday living. This is especially important now that most seventh and eighth graders are being introduced to algebra. "Where" should include words and phrases relating directly to their lives.

This approach is more efficient than rote learning. What has appeared to many as manipulation of nonsense symbols takes on meaning, something worth working into their thought processes and to be remembered.

The lessons fall under four headings:

Bridges from Arithmetic to Algebra

Estimation and Conjecture

Situational Lessons

Investigations

BRIDGES FROM ARITHMETIC TO ALGEBRA 1-17

The Standards for grades 5 thru 8 call for "presentations... so that the students can... understand the concepts of variable, expression and equation in an informal way" (p.102) ...ideas are not isolated in memory ...but are... associated with the natural language one uses and the situations one has encountered in the past. " (p.10) This syllabus is directed to that approach.

Riddles make an excellent bridge from arithmetic to algebra, introducing 1
the concepts of unknown and equation with a game with which most of them are familiar and which they enjoy. They find that algebra is something that they have used before and which should hold little terror.

The symbol of a Question Mark carries in itself the idea of an unknown, 1
while an "x" is new and strange and signifies very little in itself.

The exercises are designed to carry the student across the bridge to algebra, starting with riddles leading to equations, and the use of question marks to the use of a letter symbol.

Starting with the teacher's own riddles, the class will soon be able to present some of their own. Soon the game will get complicated and a written "language" will have to be present. Breaking the class up into small groups will provide more opportunity for each student to participate.

Algebra Before "x"s is a lesson in the same pattern. The students start 4
with the excitement of a basketball game and are led into algebraic symbols and processes.

Mixed-Up Problems and the Game of Staying Even show the need for 6
algebraic processes. The "game of staying even" leads to such formal statements as the "multiplicative property of equality". "I'll get even with you!" adds an emotional dimension to the concept.

Unwrapping the Package relates an algebraic process to a familiar operation which they have experienced. 6

"Do Nothing Numbers" mean much more to the students than "Identity Elements." After a thorough understanding is developed, they can be led into the formal nomenclature. 8

A Number with Many Disguises again starts with an informal idea leading to the formal name. This aids in blocking the common mistake of wrong cancellation. 9

Would you Tackle the Runner in Baseball? shows that different "games," multiplication, and addition, have different rules, again blocking wrong cancellation. 10

You Have to Watch It, They May not Look Alike, and More Equivalents provide training in reading symbolic expressions, each with a built-in challenge. Arithmetic and algebraic terms are shown to be handled the same way. 13

Squaring a Binomial makes very plain the identity of seemingly different expressions. Equal Expressions again strengthens the connection between symbols and numbers. It may startle your students to see a product equalling a sum in every correct case. 18

The Lead-in Approach to Word Problems "starts where the student is" with arithmetic problems in word-problem sytle, followed by the same situation changed to a problem in algebra. For instance, "If the lengths of the sides of a rectangle are six and nine feet, what is its perimeter and area?" Then: "If the length of a rectangle is ten feet and its area is thirty square feet, what is the length of the second side? What is its perimeter?" 21-22

Most algebra word problems combine two new concepts: (1) use of formulas and (2) manipulation of equations. Students who do not have a firm grasp of one or both of these concepts will find the problems very confusing.

This approach builds a smooth, strong bridge from arithmetic to algebra, from number problems to word problems. It shows that algebra is an extension of arithmetic and that the two are closely related.

Help! I'm a Prisoner in a Square Root Jail! introduces a subject 23
with humor, romance, and a lifelike situation which is a metaphor or parallel for the operation of finding square roots. Is the classroom a bit like jail?

Why Estimation? A problem open to estimation provides an excellent 27
opportunity to start where the studetns are, to involve their thinking in searching for an answer. It is their answers that are checked out. A technique which gives even the poorest student a chance to be right is to suggest a wide spread for the estimate. It the total on a column of figures is, say, 125, ask "Is it between 500 and 25?" then narrowing the range until good estimating is necessary.

The standards state.... estimation can and should be used... to foreshadow any calculation and to judge the reasonableness of any results...

..."mathematically literate" the term denotes an individual's ability to explore, conjecture and to reason logically (p.8). Conjecturing and demonstrating the logical validity of conjectures are the essence of the creative act of doing mathematics (p.81).

Triangles, Cubes, Pyramids, Circles, Spheres, and Formulas are 28
exercises in estimation and conjecture, again involving the students as they check out their answers.

How Good a Guesser Are You? requires mental estimation of the answer, 36
requiring thought before the student blindly starts following the rules for solution. Another stimulating question: What will happen if the number, 15, in the first problem, is doubled? Will the answer double? If the coefficient(s) of the variable are doubled what will happen to the answer? Conjectures should be made on these questions before the altered problems are solved.

SITUATIONAL LESSONS 37-51

Situational Lessons, as described in the Mathematics Framework for the California Public Schools, fulfill the need "to start where the students' are."

The Standards state "activities should grow out of problem situtations ...learning occurs through active as well as passive involvement in mathematics (p.9) ...apply algebraic methods to real world problem... Use and value the connections between mathematics and other disciplines.

Situational Problems provide examples of "punch line" problems, where 37
the answer is put to use.

Usable Rate Formulas show the rate formula d = rt at work with Hot 38
Rods and automobiles. These lead into special type problems where the answer is used in a lifelike situation. "Did the racer break the record?" is more exciting than "What was his speed?" The roar of the race track is evoked! The value of being able to solve the problems becomes immediately demonstrated , thereby reinforcing learning.

Which Car Has the Power? starts with simple substitution leading into 41
algebraic manipulation.

Measure an Earthquake? shows the use of exponents, especially in Southern California--right where the student lives. How Many Powers of Ten Do You Find Around You? again relates algebra to their immediate world. 42

How Far Can You see? The answer to question 2 is very likely to be "Yes, of course" which, of course, is wrong, showing the need to check one's estimates of conjectures. 44

The Instrument Was a Little Off! relates signed numbers to everyday occurences, where sometimes one adds, another time subtracts. Unusual instances of the product of two negatives are provided. 45

Turn the Air-Conditioning Down 47

INVESTIGATIONS

Investigations are the ultimate in a good mathematical lesson. Through them the student shares some of the excitement of math and develops a strong feel for a mathematical method. He or she becomes involved. Again, from the standards, they are "the essence of the creative act of mathematics."

"Learning should be guided by the search to answer questions-first on an intuitive, empirical level, then by generalizing, and finally by justifying (proving)... students can model real world phenomena with a variety of functions... represent and analyze relationships using tables, verbal rules, equations and graphs... analyze the effects of parameter changes on the graphs of functions." (p.154)

..."we see classrooms as places where interesting problems are regularly explored using important mathematical ideas."

An Investigation in Algebra Starts with a problem in numbers, a smooth transition is made to the formula $(a+b)(a-b) = a^2-b^2$. The necessity and use of letter symbols are clearly demonstrated and therefore more easily understood. 49

It is presented as a problem faced by the Arabs a thousand years ago, it becomes a historical demonstration of how mathematics is developed and how it can be used in entirely new situtations. It becomes a lesson in the processes of mathematics, inlcuding algebraic proof.

When the discussion is opened with the questions, "Which symbol makes this statement true? $576^2 \$ 575 \times 577$," the student is immediately involved. He or she is committed; a personal interest is aroused.

An Experiment with Pulleys is a complete lesson in mathematics. It starts where the student is; it is "hands-on," starts inductively and leads to a formal equation. The process is used in laboratories and engineering plants to find a mathematical formula for a physical situation. The students feel as though they are there! 51

Starting with the question, "How do you describe what happens to the weight when the hand moves up and down?" The answer in English, "The weight goes up and down." Now let's add a number line to "mathematize" the situation. The value and necessity of the algebra becomes immediately apparent.

With a number line next to the pulley, a table of values and a coordinate graph, students can be assigned as an engineer to move the pulley, a recorder on the table of values, and an analyst to draw the graph. These positions can be rotated among many students.

Using Mathematics to Investigate Thrown Balls, Gravity and Falling Water is another lesson with a more complete experience. It is much more than a series of exercises. It shows the broad use of a single formula. It demonstrates the changes caused by a change in parameters: starting speed and gravitational force. The students will see algebra when they turn on a water faucet if it is pointed out to them! 62

The investigations of Pulleys and Thrown Balls lend themselves to a problem for computers.

How Casey Got his Batting Average is fun, but it is also a solid example of applied mathematics, how a system might be developed, and the adjustments that have to be made to fit a physical situation. 70

In the spirit of the Curriculum and Evaluation standards for School Mathematics, Motivators for Algebra One aims to show beginning algebra students that the subject is an extension of arithmetic and of their own thinking. It aims to develop a strong understanding of the meaning and use of letter symbols, to show the many applications of algebra in their own lives, and to develop through investigation an understanding of mathematical processes.

These lessons were developed during the author's tenure as a teacher and department head at San Marino High School, San Marino, California and as an instructor with the Office of Continuing Education, California State Polytechnic University, Pomona.

The author wishes to express his appreciation to Dr. Pamela Clute of the University of California at Riverside and to Diane Threlkeld, Extension Officer, Office of Continuing Education at CSPU Pomona, for their encouragement and early recognition of the value of this work. Mrs. Threlkeld and her staff assisted in editing and publishing.

Louis McCreery

A STUDENT-CENTERED CLASSROOM

We tell students that mathematics is good for them, that they will need it in the future and that it is good training for the mind. Some are convinced, others just go along or reject it altogether. But how much better it would be if the student felt that it was of value right now !!!

The student will recognize a class in mathematics as a valuable experience if some of the following attributes are present.

I. A personal as well as an academic reason for students to learn a particular concept:

   A. Applications to his or her world: Riddles, thrown balls, water faucets, pulleys, gasoline pumps, races, trips, earthquakes. They help them, to quote Dr. Zoltan Deines of international fame, "TO FEEL IT IN THEIR BONES."

II. The concept is introduced in terms which are already part of the student's thinking. He or she is then led to something new. START WHERE THE STUDENTS ARE.

   A. Number riddles are solved with algebraic processes.

   B. A question mark used as a symbol for the variable already contains for the student the idea of an unknown.

III. Through questioning, conjecture, investigation, discovery, and estimation the student becomes personally involved in the answer. It is his or her ideas that are being checked out. DON'T TELL THEM, ASK THEM.

   A. Will the distance you can see from the top of a tower double if you double the height?

   B. One student's comment, "Don't show me anything I can find out for myself!"

IV. The results are personally satisfying. The student:

   A. Is successful and has a sense of growth.

   B. Knows that his work is appreciated. Even the poorest students respond to a "good here" on a bad paper. The class provides a non-threatening, nurturing atmosphere where the student feels it is alright to make mistakes.

   C. Contributes original problems or ideas.

   D. Finds he can take pride in his work and can enjoy learning.

   E. Feels more secure in this chaotic and ever-changing world for having studied mathematics, the bases of which are eternally constant.

V. There is some physical activity, games and fun.

A. The student folds paper, uses a compass and ruler, uses a pulley or throws a ball. The kinetic feeling of action is necessary to learning for some students and is helpful to all.

B. Contests, games and humor add a light touch and often involve more intense concentration than the usual classroom activity.

Lessons planned from the student's viewpoint, especially in beginning algebra, increase their effectiveness several fold. It is the student's background, his or her attitudes, abilities, and world that deserve prime consideration.

RIDDLES AND QUESTION MARKS
EQUATIONS AND VARIABLES

If a friend said, "I'm thinking of a number, add five to it and you have 12. What is the number?" You would have no trouble answering "7". If the friend said: "Four times my number is 24. What is it?" That would be easy, too. 24 divided by 4 is 6.

If the question was "twice my number plus one is 25, what is the number?" That is a bit more difficult, but you will probably take one from 25, getting 24, then divide it by two to find the answer, 12.

But if the riddle was "three-quarters of my number, minus 22 is 62," you might need to write something down to solve it. Let's look at the first problems again.

Let a question mark stand for the number we are looking for. ? plus 5 equals 12, or ? + 7 = 12.

? + 7 = 12 is a problem written in algebra, for algebra is a "shorthand" or quick way to write a problem.

4(?) = 24 and 2(?) = 12 are ways to write the next two.

The difficult riddle becomes $\frac{3}{4}(?) - 22 = 62$. Now it's laid out where we can see the problem and later in the course we will learn how to solve it.

Usually the sentences are written with an x or some other letter instead of a question mark, like x + 5 = 12.

Here are some riddles. Translate them into algebra sentences, sometimes called equations. Use either a question mark or a letter for the number you are looking for, then find the number.

1. The number added to 12 gives 15.
2. The number added to 25 gives 36.
3. The number subtracted from 21 gives 10.
4. Four times the number is 28.
5. One half the number is 15.
6. Three times the number plus 5 is 26.

Now use a letter, like "x" for these.

7. Six added to the number is 20.
8. Fifteen minus the number is 7.
9. Three quarters of a number plus 13 equals 40.
10. A number plus 5 equals twice the same number plus 3.

ALGEBRA BEFORE "X"S

You, as a beginning algebra student, will have used algebra without ever having heard the word, or perhaps heard about it with fear and trembling. Here are samples:

1. Your team has 27 points, your opponents have 35. How many points must your team make to tie?

2. What if your opponents make 7 more points?

3. (a) You have $30 toward a new portable radio costing $57.50. How much more do you need to buy it? (b) If the sale goes off in 5 days, how much do you need to earn each day to meet the price?

1. In the first problem you may have thought like this:

What number do I have to add to 27 to get 35? or
What number plus 27 equals 35? More simply,

Using a ? for "what number", ? + 27 = 35

Using n for "what number?, n + 27 = 35

Or using that x, x + 27 = 35

Then you subtracted 27 from 35:

What number is 35 - 27?: 8, so 8 points are needed.

? = 35 - 27, n = 35 - 27, x = 35 - 27

So "x" stands for a number which fits the problem, in this case 8.

2. Now, what happens if your opponents score 7 more points?

What number added to 27 will equal 35 with 7 added?

What number plus 27 equals 35 plus 7?

? + 27 = 35 + 7

Again you subtracted.

? = 35 + 7 - 27. or x = 42 - 27 or 15.

The number that fits the problem is 15.

Note that you really subtracted 27 from both sides of the equation. In problem 1:

$$\begin{array}{rrr} x + 27 = & 35 & x - 8 \\ - 27 & - 27 & \end{array}$$

3a. An equation like that in problems 1 and 2 will work for problem 3.

What amount of money do I need to add to my $30 to make $57.50?

$$? + 30 = 57.50$$
or $$x + 30 = 57.50$$
$$- 30 \quad - 30$$
$$x = 27.50$$

3b. Now, you have 5 days to earn $27.50. How much must you earn each day? 5 times some number must equal $27.50.

$$5 \text{ times } ? = 27.50$$
$$5(?) = 27.50$$
$$5m = 27.50$$

m stands for amount of money each day

Now we divide both sides by 5

$$\frac{5m}{5} = \frac{\$27.50}{5}$$

$$m = 5.50$$

This time the amount of money in one day is $5.50.

These two problems can be combined.

Five times the amount of money earned each day plus $30 will be enough to buy the stereo.

Let m be some number representing the amount of money earned each day.

$$5m + 30 = 57.50$$

or, if you let x now represent the amount of money,

$$5x + 30 = 57.50$$
$$- 30 \quad - 30$$
$$5x = 27.50$$
$$\frac{5x}{5} = \frac{27.50}{5}$$

x = $5.50, the amount to be earned each day.

In algebra, x or other letters usually are a short way to write "what number fits the problem?" Because over a series of problem its value can change or vary, it is called a <u>variable</u>.

MIXED-UP PROBLEMS AND THE GAME OF STAYING EVEN

In arithmetic, the answer is usually at the end of the problem. In algebra, it is likely to be mixed up in the problem.

arithmetic:	algebra:
15 20 11 ? answer	15 ? 11 46
2 times 3 = ?	2 times ? = 6
	The number must be 3
25 -11 ?	25 ? 14 — 11 is the number you subtract from 25 to get 14
$\frac{15}{5} = ?$	$\frac{?}{5} = 3$ — The number must be 15.

Algebra problems like these you can usually solve in your head. "Five times some number is 10. What is the number?" is easy. "Five thirds of a number minus 17 is 31. What is the number?" is not. To help with difficult problems, we need to know just what we can and cannot do with equations.

First, here is a simple game: I have ten and you have ten. I'm going to do something to my ten. You try to "stay even" with me by doing something to your ten.

I add two to my ten. What do you do to stay even? Add two, of course.

I have 12. I divide by 4. What do you do to stay even? Divide by 4.

I have 20, and I divide by 5. What do you do to stay even?

Try this game with a fellow student.

We must preserve <u>equality</u> to get the right solution set for the answer.

You can add or subtract the same number from each side of an equation, and not change the answer or solution set. This is the ADDITIVE PROPERTY OF EQUALITY. You can multiply or divide each side of an equation by the same thing and not change the answer or solution set. This is the MULTIPLICATIVE PROPERTY OF EQUALITY.

CAN YOU COME FROM BEHIND?

I have 10, you have 2.

me you
10 > 2

Using the same rules for equalities, adding or multiplying each side by the same number, can you get ahead of me? There is a way!

MATHEMATICS IS ABOUT LIFE

It was a great day when our astronauts landed on the moon. It was a great day for mathematics, for without it the mission could not have been conceived or carried out. Mathematics is one of the main bases of our life and our progress. Without it, we would still be in the stone age, bartering for goods.

With it, we sent a probe to Uranus and beyond; without it we would still be hunting with bows and arrows. Without mathematics, no high finance or football scores would be possible. Anyone asking "how much" or "how many" would be answered with such words as "heap."

Open your eyes to see how great it is.

UNWRAPPING THE PACKAGE

Louis McCreery

In a problem like 3/4x + 15 = 75, finding the number value of "x" that makes the sentence true is like unwrapping a package. With a package, you must be careful not to break what is inside. With an equation, you must follow the rules to find the right value for "x". You "unwrap" it from the numbers around it.

For instance, the first "wrapping" in 3/4x + 15 = 75 is the number +15. To "unwrap" it, we subtract 15 from <u>both</u> sides of the equation. You must do the same thing to each side to keep them even and not change the value of "x".

Step I

$$\begin{array}{r} 3/4x + 15 = 75 \\ -15 \quad -15 \\ \hline 3/4x = 60 \end{array}$$

Now for the second "unwrapping", multiply both sides by 4.

Step II

$$4\left(\frac{3x}{4}\right) = 4(60) \qquad \frac{4}{4} = 1$$

and can be dropped in a multiplication problem

therefore $3x = 240$

Now divide both sides by 3.

Step III

$$\frac{3x}{3} = \frac{240}{3}$$

therefore $x = 80$

To see if you have the right answer, does 3/4(80) plus 15 equal 75?

Yes, 60 plus 15 does equal 75.

Here are some one-step or one-wrapping problems:

1. $x + 15 = 60$ 2. $x - 15 = 60$ (You add 15 to both sides in problem 2.)

3. $12 + x = 75$ (Add the opposite of +12, not the x) Keep the unknown, the x, on the left side.

4. $6x = 12$ 5. $\frac{x}{4} = 16$ 6. $\frac{x}{7} = 1.5$

see Step II Step III

Two-step problems: Remember, addition or subtraction comes first.

7. $\frac{x}{5} + 10 = 30$ (I,III) 8. $3x + 20 = 60$ (I,II) 9. $4x - 20 = 40$

10. $\frac{4x}{5} = 110$

Three-step problems:

12. $\frac{8x}{5} - 15 = 45$ (I,II,III) 13. $20 + \frac{7x}{2} = 69$ 14. $\frac{3x}{7} + 11 = 20$

15. $\frac{4x}{9} - 10 = 13$

15. answer
51 3/4

The rules for solving an algebraic equation are like the directions on a treasure map. You have to be careful to follow them exactly to get to the buried treasure--the answer.

DO NOTHING NUMBERS

Louis McCreery

In some cases, certain numbers have no power to change the answer to aproblem. They do nothing. For instance, 3 + 0 = 3. 3 + 2 = 5, but 3 + 2 + 0 = 5 also. The addition or subtraction of zero (0) does not change the answer to a problem, leaving it "identical" to what it was.

So zero is called the IDENTITY ELEMENT FOR ADDITION AND SUBTRACTION. When added or subtracted in a probm, 0 leaves the answer just as it was, identically the same. But if you multiply by zero, its power destroys the value of the answer 3 · 0 = 0. 3 · 4 = 12, but 3 · 4 · 0 = 0.

Is there a "do nothing" number, an identity element for multiplication? In this equation 3 · ? = 3, what number does not change the value 3? 3 · 4 · ? = 12. 9 · 7 · ? = 63. You have probably guessed it. The number is one (1). But the number 1 does have the power to change the answer in 3 + 4 = 7, for 3 + 4 + 1 = 8, not 7.

Multiplying or dividing by 1 makes no change in the answer to a problem. Try these:

1) 5 · 6 = _____ 2) 5 · 6 · 1 = _____ 3) 5 · 6· 0 = _____

4) 5 + 6 = _____ 5) 5 + 6 + 0 = _____ 6) 5 + 6 + 1= _____

7) 20 ÷ 2= _____ 8) 20 1 = _____ 9) 20 ÷ 20 = _____

THE NUMBER ONE IS THE IDENTITY ELEMENT FOR MULTIPLICATION AND DIVISION.

A NUMBER WITH MANY DISGUISES

Louis McCreery

The number one can be written in many ways. For instance:

$$1,\ I,\ \frac{2}{2},\ 3\text{-}2,\ \frac{100}{100},\ \frac{a}{a},\ 4/4,\ \frac{x^3}{x^3},\ \frac{x-2}{x-2},\ 75 \div 75$$

Any number divided by itself gives an answer of one. (1)

A numeral for one can be replaced with another numeral for one. $\frac{2}{2}$ can be replaced with 1. $\frac{4 \cdot 2}{2}$ can be written $2 \cdot \frac{2}{2}$ and this becomes $4 \cdot 1$, or just 4. We can drop the 1, for being the Identity Element for Multiplication, it makes no difference in the result.

But we can't drop the 1 $\frac{2}{2}$ here.

The addition "game" has different rules than the multiplication "game". As you wouldn't tackle the runner in baseball, you wouldn't multiply by zero or add 1 in an addition problem to keep the answers the same.

List the fractions which are completely multiplication or division problems. there are exactly 4.

1) $\frac{5 + 10}{5}$ 2) $\frac{5 \cdot 10}{3}$ 3) $\frac{x + 15}{x}$ 4) $\frac{13a}{13}$

5) $\frac{3^2 + 4 \cdot 5 + 10}{3^2}$ 6) $\frac{3^2 (4 \cdot 3) \cdot 10}{3^2}$ 7) $\frac{x + 3}{x}$

8) $\frac{x \cdot x^2 \cdot y}{x}$ 9) $\frac{x \cdot x^2 + y}{x}$

Can you find a disguised numeral for 1, the Identity Element for Multiplication, in those you lsited? Or can you find a number divided by itself?

WOULD YOU TACKLE THE RUNNER IN BASEBALL?

Louis McCreery

A. 1. Is $\frac{3 + 10}{3} = \frac{13}{3}$? yes _____ no _____

2. Does this dividing out (cancellation) of the 3's still give a value of 13/3? $\frac{\cancel{3} + 10}{\cancel{3}}$ yes _____ no _____

3. Is this dividing out proper? yes _____ no _____

4. If you think so, put in 3 for x and divide out. $\frac{x + 10}{x}$

Did you still get 13/3? yes _____ no _____

5. $\frac{3^2 + 4 \cdot 3 + 10}{3^2} = \frac{31}{9}$

Is it right to divide out the three squareds (3^2)?
yes _____ no _____

6. If you do, do you still get 31/9? yes _____ no _____

Cancelling out is an operation for <u>multiplication</u> and <u>division</u>. You can't use it when <u>addition</u> is involved, as it is in problems 1-6, any more than you can <u>tackle the runner in baseball</u>!

7. Does $\frac{3 \cdot 10 \cdot 5}{5} = \frac{150}{5}$ or $\frac{30}{5}$

Does $\frac{3 \cdot 10 \cdot \cancel{5}}{\cancel{5}} = 30$ Yes

Cancelling by finding a numeral for one is correct in this totally multiplication problem. 5/5 = 1 and 1 is the IDENTITY ELEMENT for multiplication and division. It does not change the answer in a completely multiplication or division problem.

B. List the fractions whose numerators are a group of just factors (multipliers) with no outside addends (things to be added). There are only 4.

1. $\frac{7 \cdot 10 \cdot 5}{25}$ 2. $\frac{7+9+5}{25}$ 3. $\frac{7 \cdot 9+5}{25}$

4. $\frac{x \cdot x \cdot y}{x}$ 5. $\frac{x+x+y}{x}$ 6. $\frac{x \cdot x+y}{x}$

7. $\frac{x^2+7x+10}{x}$ 8. $\frac{x(x^2+7x+10)}{y}$ 9. $\frac{x+(x^2+10x+7)}{x}$

10. $\frac{(x+5)(x+2)}{x+2}$ 11. $\frac{(x+5)+(x+2)}{x+2}$ 12. $\frac{x^2(x+5)+(x+2)}{x+2}$

Binomials like a-2 can be multipliers when enclosed in parentheses (a-2) or when standing alone.

In the problems you lsited, make a check next to those which have a common factor in the numerator and denominator, then cancel out this numeral for one.

C. If you can, change the numerators and denominators of the fractions below from a group of addends to a group of factors (multipliers). <u>This is what you do when you factor</u>. If possible, reduce the fraction by finding a numeral for one.

1. $\frac{3x+3}{3}$

2. $\frac{4x+4}{3x+3}$

3. $\frac{x^2+xy}{x}$

4. $\frac{x^2+10x+7}{x+2}$

5. $\frac{3x+1}{3}$

6. $\frac{x^2+2x}{x+2}$

7. $\frac{x^2-7x+10}{x+2}$

8. $\frac{4x^2-1}{2x+1}$

9. $\frac{16x^2-4}{4x-4}$

10. $\frac{17a^2-68}{a-2}$

11. $\frac{a^2+2ab+b^2}{a^3-ab^2}$

YOU HAVE TO WATCH IT

You have to keep your eyes open when you read things written in algebra. Some look alike, like two coins from the same mold. $4x^2-5$ and $4x^2-5$ are plainly alike.

Some expressions look alike, but are different in some small detail, like two Roosevelt dimes of different dates. $4x^3-5$ and $4x^2-5$ are different only in the exponent of x. You could miss that detail easily.

Some don't look alike, but stand for the same number. Two nickels and one dime don't look alike, but are each worth ten cents. 5+5 and 10 stand for the same number.

EXPRESSIONS OR FIGURES EXACTLY ALIKE

Look carefully at each pair. If they are alike in each detail, write the number of the pair in the answer column. There are just enough spaces for the right answers. LOOK SHARP!

1. $3+3,\ 3+3$

2. $x+2,\ x+2$

3. $5-3,\ 5+3$

4. $a^2,\ 2a$

5. $d+5,\ d+5$

6. $\frac{x+5}{2}, \frac{x+5}{3}$

7. $3x^2y,\ 3x^2y$ _____

8. $2x^3y,\ 2xy^3$ _____

9. $y-3=0,\ y-3$ _____

10. $7.26,\ 72.6$ _____

11. $5x^3y^4,\ 5x^4y^3$ _____

12. $\frac{x^2+y^2+z^2}{xyz}, \frac{x^2+y^2z^2}{xyz}$

13. $x^2y^2z,\ x^2y^2z$

EXPRESSIONS OR FIGURES EXACTLY ALIKE, P.2

1.	2+2	2+2	18.	x-3	x+3
2.	3+2	5	19.	x-3	3-x
3.	5.14	5.14	20.	x-3	-3+x
4.	5.78	57.8	21.		
5.	x	x			
6.	x^2	x^2	22.	2x+5	2x+5
7.	x^2	x^3	23.	2x+5	2x+5=0
8.	2x	2x	24.	(a+2)(a+5),	(a+2)(a+5)
9.	3x	x^3	25.	(x+3)(x+7),	(x+3)(x+7)
10.	x+5	x+5	26.	7x11	7+11
11.	$\frac{x+5}{7}$	$\frac{x+5}{9}$	27.	x+x+5	2x+5
			28.	$2x^3+x+1$	$2x^2+x+1$
12.	$3x^2y$	$3x^2y$	29.	7<5x	7>5x
13.	5.0132	5.0123	30.	-9+6+7-1,	-9+6+7-1
14.			31.	a+5(a-7),	(a+5)(a-7)
15.					
16.	x	x+5			
17.	x-3	x-3			

THEY MAY NOT LOOK ALIKE !!!

Two expressions which stand for the same number may or may not look alike. Four quarters don't look like a dollar bill, but they are worth just as much. Four quarters and a dollar bill have the same value. $7+5$ stands for 12, but doesn't look like 12. $x+x$ stands for the same number that $2x$ does.

$4/8$ and $1/2$ are different ways of writing the same number. .5 is another. Expressions which stand for or represent the same number are called <u>equivalent</u> expressions. $4/8$, $1/2$, and .5 are members of the <u>set</u> of <u>equivalent expressions</u> for the number one-half.

If the two members of each pair below are <u>equivalent</u>, if they stand for the same number, put an equal sign between them. For example, $2+2=4$.

1.	$3+3$	6	2.	5-2	7-4	3.	$6+2$	12-2
4.	5.00	5	5.	3 dimes, one quarter			eleven nickles	
6.	5 nickels & 7 cents			3 dimes & 2 cents				
7.	10 dimes & 6 nickels			5 quarters		8.	$x+3$	$x+3$
9.	$3+x$	$x+3$	10.	$a+a$	$2a$	11.	$a+a$	a^2
12.	$3a+a$	$4a$	13.	$x \cdot x$	x^2	14.	$x+x$	x^2
15.	7^2	7^2	16.	$5 \cdot 7 \cdot 3$	$5 \cdot 3 \cdot 7$	17.	$5ab$	$5ba$
18.	x^2yz	xy^2z	19.	$\frac{9}{6}$	$\frac{3}{2}$	20.	$\frac{100}{50}$	2
21.	$\frac{60}{30}$	$\frac{1}{2}$	22.	$\frac{7}{11}$	$\frac{14}{21}$	23.	$\frac{2x^2}{4}$	$\frac{1}{2}x^2$
24.	$\frac{12a^2}{3}$	$4a^2$	25.	$\frac{x^2}{x}$	x	26.	$\frac{x}{x^2}$	$\frac{1}{x}$

27. $\frac{a}{a^2}$ a 28. x + x + x 3x 29. m + m + m m^3

30. x - 3 3 - x 31. x - 3 -3 + x 32. 1 ft. 2 in. 12 in

33. 1 yd. 1 ft. 1 in. 4 ft. 1 in. 34. 1 lb. 4 oz. 15 oz.

35. 15 in. 15 cms. 36. (x + y = 5) (y + x = 5)

37. (x+y = 5) (xy = 5) 38. (a+5)(a-3) (a+5)+ (a - 3)

39. (a +5)(a - 3) a+5(a - 3) 40. $\frac{25 - a^2}{b}$ $\frac{a^2 - 25}{b}$

41. 17 - 21 27 - 31 42. 29 - 25 25 - 29 43. $a^2 + b^2 + c^2$ $a^2+b^2c^2$

44. $\frac{a(b+c)}{a}$ b + c

There are 27 pairs which are equivalent.

AT THE CHECK-OUT COUNTER

The line at the checkout counter at a special sale where 20% was discounted from each customer's total was moving rather slowly. The checker added the total, calculated the 20%, then subtracted it from the total to find the final cost. The manager made a simple suggestion to the clerk and the line moved twice as fast. What was the suggestion for the calculations?

Answer: As 100% minus 20% is always 80%, take 80% of each total-just one step.

MORE EQUIVALENTS

PART I

Similar terms have the same bases (letters) and the same exponents for each base.

$4a^2x^3$ 2 and 3 are exponents
a and x are bases

For the terms to be similar, the "coefficient" 4 need not be the same.

NOTE: $\frac{1}{y}$ is the same as y^{-1}. a^2b is similar to ba^2

Circle the terms that are similar to x^2y. (Hint: there are 7 such terms.)

1) $\frac{x^3}{y}$ 2) x^2y 3) y^2x 4) yx^2 5) $\frac{x^2}{y}$

6) $3x^2y$ 7) y^2x^2 8) $4x^2y$ 9) $5yx^2$ 10) xy

11) $-2x^2y$ 12) xy 13) x^2y^1 14) x^2y^{-1} 15) $\frac{1}{x^2y}$

Put a cross through the terms which cannot be combined by addition with x^2y. (In the above exercises, all terms should be either circled or crossed, but not both.)

PART II

Below, circle the expressions which indicate at least one addition or subtraction, possible or not. (There are 10).

1) (2a-a) 2) (3b-a) 3) (3b) (-a)

4) (abc+bc) 5) (3b+4b) 6) $(3b^2+4b)$

7) (3b-7b) 8) $5(+16a^2)$ 9) $x^2+7x+12$

10) $3x^25x^2+8$ 11) $(3x^3y+4xy+4xy^3)$

12) $(a^3b)(-a^3b)$ 13) $(a^3b)-(a^3b)$

Now perform the additions which are possible, combining only similar terms. Write the answers below with the number of the problem. (There are 4 possibilities.)

ON SQUARING A BINOMIAL OR

X IS A NUMBER

1. Find the value of:

let x be: 1	$(x+2)(x+2)$	x^2+4x+4	$(x+2)^2$
-1			
0			
2			
-2			
5			
$-\frac{5}{2}$			

2. Are the answers alike in each column? _____

3. Why are the answers alike? _____

4. What do you notice about the answers? _____

_____

5. Why do they <u>have</u> to be perfect squares? _____

_____

6. $(9x - 5)^2 = 81x^2 - 90x + 25$ of this equation

Let $x = 4$. Would you use the left side or the right side/to evaluate the expression? _____

How many multiplications would it take for (a) the right side? (b) the left side? (a) _____ (b) _____

What Does "Equal Expressions" Mean?

If two algebraic expressions are equal, the numbers they produce will be equal. Two equal expressions connected with an equal sign make up an "identity."

We can multiply two binomials to form an identity. $(x-2)(x+3)$ can be broken down to:

$$x(x+3) + -2(x+3)$$

$$x^2 + 3x - 2x - 6$$

$$x^2 + x - 6$$

Does $(x-2)(x+3)$ really produce the same numbers as $x^2 + x - 6$?

$(x-2)(x+3) \stackrel{?}{=} x^2 + x - 6$. We'll substitute a number, say 4, for x.

$$(4-2)(4+3) \stackrel{?}{=} 16 + 4 - 6$$

$$2 \times 7 \stackrel{?}{=} 14$$

$14 = 14$ Yes, we have an identity.

1. Does substitution of -4 produce the same number on each side?
2. Here are two possible identities, differing only by changes in sign. Only one of them is true.
   a. $(x+2)(x+4) \stackrel{?}{=} x^2 + 6x + 8$. b. $(x-2)(x+4) \stackrel{?}{=} x^2 - 6x - 8$
   Find the false one first by substituting a number, then prove it by showing that the binomials do not produce the right-hand side when multiplied together.

Some of the following are true identities, some are not. Try several values for x to see if equal numbers result. They may be proved by multiplying the binomials on the left side to see if the right side is produced or factoring the right side to see if the left side results.

3. $(2x-1)(x+3) \stackrel{?}{=} 2x^2 + 5x - 3$

4. $(2x+3)(x-2) \stackrel{?}{=} 2x^2 - x - 6$

5. $(x-4)(x+12) \stackrel{?}{=} x^2 - 16x + 48$

6. $(5x+11)(3x-10) \stackrel{?}{=} 15x^2 - 17x - 110$

7. $(y+7)(y-7) \stackrel{?}{=} y^2 + 49$

LEAD-IN PROBLEMS FOR ALGEBRAIC WORD PROBLEMS

1a. Jones's salary is $1250 a month. How much does he earn in a year?

b. In nine months Johnson earns $5670. What does he earn each month?

2a. Smith travelled 200 miles by auto, then eight times as far by plane. How far did he go by plane? How far did he go altogether?

b. Mrs. Smith travelled a certain distance by car and eight times as far by plane. The total distance was 630 miles. How far did she go by car? By plane?

3a. Brown travelled by car for one hour at 50 mph, and for two hours at nine times the speed of the car by plane. What was the speed of the plane? How far did he go by car? By plane? How far altogether?

b. Mrs. Brown travelled by car for two hours and three hours by plane at ten times the speed of the car. The total distance travelled was 992 miles. How do you represent the speed of the car? The distance the car travelled? The speed of the plane? The distance the plane travelled? What does the problem say about the total distance? Make an equation. Find the speed of the car. Find the speed of the plane.

4a. What is the sum of a number and its square if the number is 9? If it is 17? If it is -7?

b. What is the square of 19 if the square plus 19 is 380?

*c. What is a number if the sum of its square and the number is 72? Is 20? 132?

5a. A plane leaves the airport at noon flying at 350 mph. How far will it fly in 3 hrs.? 5 1/2 hrs.? 4 hrs. and 40 min.?

b. How long will it take to fly 1050 miles? 1225 miles?

c. If it flew 750 miles in one hour 45 minutes, how fast was it flying?

6a. One plane leaves the airport going west at 350 mph. Another leaves at the same time going east at 425 mph. How far apart will they be in 3 hrs.? 4 hrs. and 30 min.?

b. In how many hours will they be 3100 miles apart?

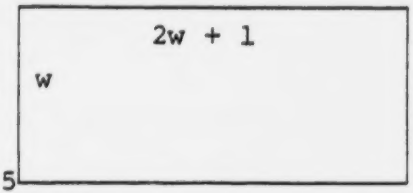
c. A plane left at the rate of 375 mph. Another left at the same time going in the opposite direction. If they were 2000 miles apart in three hours, what was the rate of the second plane?

7a. The length of a rectangle is twice its width. Its width was 4 yds. What is the length of the rectangle? Its perimeter? Its area?

b. The length of a rectangle is twice its width. If the perimeter is 36 yrds., what is the width, the length, the area?

ans. w = 6 yds.

8a. A set of rectangles all have their lengths one foot longer than twice the width. If the width of one rectangle was 5 ft., what was the length? The perimeter? The area?



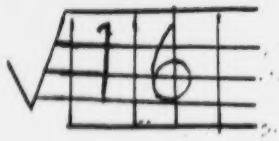
length $2(5) + 1 = 11$
perimeter = $2 + 2w$
$22 + 10 = 32$ ft.

area = lw
$5 \times 11 = 55$

b. Another rectangle of the same set has a width of six feet. Fill in the table below for this and other rectangles of the set.

width	length	perim-eter	area
6'			
10'			
	15'		
	24'		
		50'	
w	2w + 1		6 sq. ft.*
			15 sq. ft.*
			28 sq. ft.*

*Need quadratics for solution

HELP!

HELP!

HELP - - I'M A PRISONER IN A SQUARE ROOT JAIL ! !

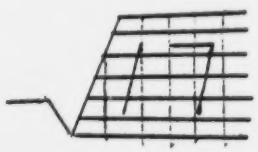
How can I get out? How can I get out?

YOU are lucky, sweet 16. YOU are a perfect square.

YOU can go outside, BUT only as your SQUARE ROOT 4.

$\sqrt{16} = 4$

* *



I'm her boy friend 17. How about me?

No luck, Buster, YOU are NOT a perfect square.

You are irrational; not crazy, just irrational.

* *

$\sqrt{2 \times 9} = \sqrt{2} \times \sqrt{9}$

Please, how about me? I'm 18.

It doesn't look too good.

But I can split up into 2x9.

9 gets out as 3. 2 stays behind.

(tears) So long old friend!

$$\sqrt{\frac{1}{4}}$$

I'm a perfect square, but I'm down here in the basement! OK, but your square root will be in the basement or denominator when you get out!

$$\sqrt{\frac{1}{4}} = \frac{1}{2}$$

* *

I'm $\sqrt{\frac{1}{8}}$

You can't leave me here! The law says no prisoners in the basement! You'll have to make my denominator, 8, a perfect square. Multiply top and bottom by the same number, which makes the denominator a perfect square.

$$\sqrt{\frac{1}{8}} = \sqrt{\frac{1\times 2}{8\times 2}} = \sqrt{\frac{2}{16}} = \frac{1}{4}\times\sqrt{2}$$

The multiplier is 6.

$$\sqrt{\frac{1}{6}} = \sqrt{\frac{1\times 6}{6\times 6}} = \sqrt{\frac{6}{6^2}} = \frac{1}{6}\sqrt{6}$$

These prisoners are in bad shape. Two or more things have to happen. Take care of the denominator first.

$$\sqrt{\frac{8}{27}} = \sqrt{\frac{8\times3}{27\times3}} = \sqrt{\frac{24}{81}} = \frac{1}{9}\sqrt{24}$$

24 has a factor which is a perfect square. The square root of 4 is 2.

$$= \frac{1}{9}\sqrt{4\times6}$$

$$= \frac{2}{9}\sqrt{6}$$

* * * * * * * * * * * * * * * * *

Sample: $\sqrt{\frac{a^3}{2}} = \sqrt{\frac{a^3\times2}{4}} = \frac{1}{2}\sqrt{2a^2\times a} = \frac{a}{2}\sqrt{2a}$

* * * * * * * * * * * * * * * * *

a. $\sqrt{\frac{32}{3}} =$ _____

b. $\sqrt{\frac{32}{27}} =$ _____

c. $\sqrt{\frac{27}{2}} =$ _____

d. $\sqrt{\frac{a^2x}{8}} =$ _____

e. $\sqrt{\frac{15}{2}} =$ _____

answers

a. $\frac{4}{3}\sqrt{6}$

c. $\frac{3}{2}\sqrt{6}$

Summary questions:

1. When do you break the number into factors?

_____

2. When do you multiply to make a perfect square?

_____

_____

answers: 1. When there is no denominator under the radical, and the number has a perfect square factor.

2. When you have a denominator and must remove it. You multiply the numerator and the denominator by a number which makes the denominator a perfect square.

SOME QUESTIONS ON THIS GREAT DRAMA

1. Who gets out easily (like sweet sixteen)? _____

_____

Fill in the blanks:

$\sqrt{64} =$ ___, $\sqrt{81} =$ ___, $\sqrt{a^2} =$ ___, $\sqrt{d^4} = d^2$

$\sqrt{x^{16}} =$ ___, $\sqrt{x^6} =$ ___, $\sqrt{121} =$ ___, $\sqrt{a^2c^2} =$ ___, $\sqrt{2025} =$ ___

2. Who can be factored and the square root of a part gets out (like 18)? _____

$\sqrt{8} = 2\sqrt{2}$, $\sqrt{27} =$ ___, $\sqrt{32} =$ ___, $\sqrt{200} =$ ___

$\sqrt{72} =$ ___, $\sqrt{a^3} = a\sqrt{a}$, $\sqrt{x^3} =$ ___, $\sqrt{a^2x} =$ ___

3. Whose square root gets out, but lands in the "basement" or denominator? _____

$\sqrt{\frac{1}{9}} = \frac{1}{3}$, $\sqrt{\frac{1}{16}} =$ ___, $\sqrt{\frac{1}{25}} =$ ___, $\sqrt{\frac{1}{a^2}} = \frac{1}{a}$

$\sqrt{\frac{1}{x^2}} =$ ___, $\sqrt{\frac{1}{x^4}} =$ ___, $\sqrt{\frac{1}{a^2b^2}} =$ ___

4. <u>Who has to get out</u>? And how do you change him so he can? His square root will be a denominator outside!

_____

$\sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}$, $\sqrt{\frac{1}{3}} =$ ___, $\sqrt{\frac{1}{27}} =$ ___, $\sqrt{\frac{1}{a}} = \frac{1}{a}\sqrt{a}$

$\sqrt{\frac{1}{x}} =$ ___, $\sqrt{\frac{1}{a^3}} =$ ___, $\sqrt{\frac{5}{6}} =$ ___

Watch for this tricky bit: in $\sqrt{\frac{1}{2}}$ you multiplied top and bottom by 2 under the <u>radical</u>, which means you multiplied by the square root of 2 or $\sqrt{2}$.

In $\frac{1}{\sqrt{2}}$ you multiply by $\sqrt{2}$. Also:

$\frac{1}{\sqrt{2}} = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$

$\frac{1}{\sqrt{3}} =$ ___ $\frac{1}{\sqrt{7}} =$ ___

WHY ESTIMATION?

1. Estimation gives the student a reason to take a careful look at the problem, not to blindly proceed to use the concept in question.

2. The student is checking out his or her estimate when he or she solves the problem.

3. It develops a sense of the rightness of an answer. In this computer age, the ability to recognize errors is becoming more important than the ability to compute. "It just doesn't look right." If one has an approximate idea of what the answer should be, the errors are more apparent.

4. Estimates and approximations are useful in planning.

GOOD QUESTIONS

Is mathematics an invention or discovery?

Is division associative? Commutative?

Tension breakers for the upper left-hand corner of a test:

How did the little triangles get to school? On a rhom-bus.

Knock, knock. Who's there? Poly. Poly who? POLYNOMIAL.

Algebraic symbols for:

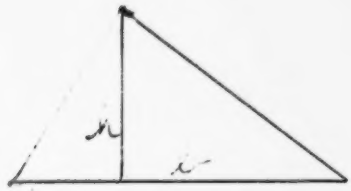
A stomach ache: $\frac{1}{8\pi}$ *Extreme happiness:* x^c

These are polynomials: *3 yards, 2 feet, 4 inches*
1 pound, 4 ounces
7½

To parody a TV commercial: "You can't teach them anything,

THEY HAVE TO LEEAARRNN IT."

TRIANGLES



HOW MANY TIMES LARGER?

How many times larger will the area of a triangle be if you double the base? your guess: _____

The formula for the area of a triangle is

$$A=\frac{1}{2}bh,$$

where b is the number of units in the base, and h is the number of units in the height.

1. Find the area if b is 5 ft. and h is 12 ft. _____

2. In problem 1, what's your guess as to what happens if the base, 5 ft., is doubled to 10 ft. and the height remains the same?

   Will the area double? Yes _____ No _____

   Find the new area. _____

   Did it double? Yes _____ No _____

3. What will happen if both the base, 5 ft., and the height, 12 ft., are doubled? Will the area be twice as large? _____

   Three times as large? _____ Four times as large? _____

   What is the new area? _____ Was your guess correct? _____

4. If the base, 5, and the height, 12 are both multiplied by 3, what is your guess as to how many times larger the area will be? 3 times as large? _____ 6 times as large? _____

   9 times as large? _____

   a. Find the new area.

   b. Fill in the rule:
      If both the base and height of a triangle are multiplied by n, the area will be _____ times larger.

   c. Check your rule out on the 5 x 12 triangle by multiplying each side by 4, and finding the area. _____

   d. Now multiply by 5. _____ Was your rule correct? ____

5. For practice in decimals, find the area of a triangle with a base of 6.7 cm and a height of 2.3 cm. _____

6. The dimensions of a triangle are 7 3/4 inches and 5 5/8 inches. About what will the area be?

   about 36 sq. in.? _____

   about 20 sq. in.? _____

   about 40 sq. in.? _____

   Find the area. _____

7. Make a guess as to how many square inches would be added to the area of the 5x12 triangle if the sides in problem 1 were increased to 5.2 inches and 12.2 inches. _____

   Find the new area. _____

   How much did the area increase? _____

AT A SELF-SERVE GAS STATION

A common situation these days is to decide how much to prepay the cashier at a gas station so that you can fill your tank. You usually prepay in dollar bills and pick up any change.

1. Your car needs about 11 gallons at 76¢ a gallon. How many dollars should you prepay the cashier to fill the tank? _____ If you put in exactly 11 gallons, how much change should you get? _____

2. Your tank holds 16 gallons and is about $\frac{1}{4}$ full. At 72¢ a gallon, how many dollars should you give the cashier so you would be sure of a full tank? _____ If the cost were 75.9¢ per gallon? _____ For estimates, round to the nearest dollar.

3. Your gas gauge reads about $\frac{1}{2}$ way full for a 16 gallon tank. You filled it up at a cost of $5.74 with gas at 82¢ per gallon. How accurate was your gauge? _____

4. Your gauge reads about $\frac{1}{2}$ way between $\frac{1}{2}$ full and full on a 12 gallon tank. At 74.9¢ per gallon, what should you give the cashier? _____

Answers 1) $9, 2) $9, 3) it read 1 gallon too much. The car took only 7 gallons, the gauge read 8, 4) $3.

WHAT HAPPENS TO THE VOLUME OF A CUBE?

What's your guess? Does the number of square inches on the sides of a cube increase faster than the number of cubic inches contained in it?

yes _____ no _____

The formula for the area of the sides of a cube is

$$A = 6s^2$$

The formula for the volume is $V = s^3$.

1. To help you check your guess, find the area and volume for:

		area	volume
a.	$s = 1$ in.	_____	_____
b.	$s = 1.6$ in.	_____	_____
c.	$s = 2$ in.	_____	_____

d. When you doubled the side from one inch to two inches, how many times larger was the area of the sides? _____

How many times larger was the volume? _____

2. Now find the area and volume for:

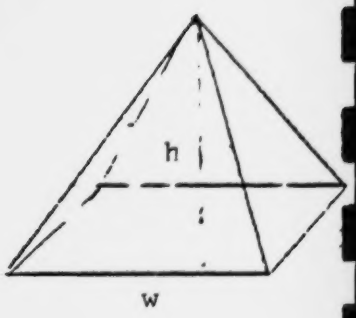
		area	volume
a.	$s = 3.2$ in.	_____	_____
b.	$s = 6$ in.	_____	_____
c.	$s = 10$ in.	_____	_____
d.	$s = 12$ in.	_____	_____

e. Using the answer in 1b, can you find the answers to 2a with only <u>one</u> multiplication?

f. The answers for d. $(s = 12)$ give you the number of square inches in a square foot and the number of cubic inches in a cubic foot!

3. How many cubic feet in a cubic yard?

HOW ABOUT PYRAMIDS?



How many times greater will the volume be if you double the length?

your guess: _____

1. The formula for the volume of a pyramid is $V = \frac{1}{3} \ell wh$. "ℓ" is the length of the rectangular base, "w" is the width of the base, and "h" is the height of the pyramid.

a. Find the volume if $\ell = 4'$, $w = 7'$ and $h = 9'$. _____

b. Double ℓ to 8', leaving $w = 7'$ and $h = 9'$. What is the new volume? _____

How many times greater is the first volume than the second? _____

c. What will happen if you double both the length and the width?

your guess: _____

Find the volume now. _____

d. What will happen if you double the length, width, and height?

your guess: _____

Use 8", 14" and 18" to find out. _____

How many times greater was this new volume? _____

e. Make a guess as to how much the volume of the pyramid will increase if the sides are increased to $\ell = 4.2$, $w = 7.2$, and $h = 9.1$. _____

Find the new volume _____

How much did it increase? _____

CIRCLES & SPHERES

1. The formula for the circumference of a circle is $C = 2\pi r$, and the area is $A = \pi r^2$; for $\pi = \frac{22}{7}$ and r is 7 in., find C and A.

   C _____ A _____

   b. Now double the radius so r =14, and find C and A.

   C _____ A _____

   c. Which increases at a faster rate, C or A? _____

   d. Find C and A if $\pi = 3.14$ and r= 5.2. C _____ A _____

2. The formula for the volume of a sphere is $V = {}^{4}/_{3}\,\pi r^3$.

   a. Find V if r =3', $\pi = \frac{22}{7}$ (answer in fractions). _____

   b. If you double the radius to 6", how many times larger will the volume be? Guess. _____

   c. Calculate the volume of a sphere with a 6' radius to find out. _____ cu. ft.

   d. How many times larger was the second sphere than the first? _____

3. How much will the volume increase if you increase the radius from 2 to 2.2? Use $\pi = 3.14$. your guess _____

   answers to 2 decimals.

   a. volume r =2 _____

   b. r =2.1 _____

   c. amount of increase to nearest cubic inch _____

   d. Now if you triple the radius from 2 inches to 6 inches, how many times greater will the volume be? your guess _____

   Use your answers to 3a and 2c to check your guess.

   e. Fill in the general rule.
   If the radius of a sphere is multiplied by n, the new volume will be _____ greater.

FANCY FORMULAS

1. The sum of numbers in order starting with 1 is found by this formula:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

n is the last number to be added.

Find n for the first seven numbers by:

a. adding them up. _____

b. by the formula. _____

c. How much larger will the sum be for n = 8? _____

for n = 10? _____

2. The sum of perfect squares in order is given by the formula:

$$1 + 4 + 9 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

a. Find the sum for the first 7 perfect squares by the formula. _____

b. What will be the sum of the first 9 squares? _____

3. The number of degrees in the interior angle of a regular polygon is given by the formula: $\text{INT}\angle = 180^\circ \left(\frac{n-2}{n}\right)$

n = number of sides

a. The figure has 8 sides. What is the size of the angle? _____

b. What is your guess as to the size of an angle of a polygon with 9 sides? your guess _____

Size by the formula _____ Amt. of increase _____

c. What is your guess for n=10?

Less than 145°? _____ Greater than 145°?_____

Equal to 145°? _____ Check your answer by the formula.

d. As the number of sides increase, how large can the angle become? _____

WHAT GOOD IS THIS STUFF?

No, most of you will not be called upon to factor a four-power expression or a five-term polynomial. But you will have to follow a contract carefully and note the details of an arithmetic problem like a complicated bill of sale.

Yes, a complicated algebra problem can be confusing and frustrating. But, like strenuous physical exercise, it stretches your mental muscles. You get a workout in following directions, a skill which is most useful in such widely different areas as building a model airplane to following a car repair manual. There is a "how to" book in every field.

You learn to watch for details: x^2 and 2x are different, just as $10.00 and $1000 are different.

If a lesson is frustrating, it demonstrates your need to learn to exercise more care and to develop patience.

HOW GOOD A GUESSER ARE YOU ??

Make a guess as to where the answer will fall before you write anything. Then solve.

1. $2x+1/3x = 15$ — $x>5?$ _____ $x<5?$ _____ answer ___

2. $7x-3x = 6x+2$ — $x>5?$ _____ $x<5?$ _____ answer ___

3. $2/3x+ \cdot 11$ — $x>5?$ _____ $x<5?$ _____ answer ___

4. $3/4x+50 = 72$ — $x>5?$ _____ $x>5?$ _____ answer ___

5. $17y-54 + 2y+10$ — $y<10?$ _____ $y>10?$ _____ answer ___

6. $15y+46 = y-11$ — $y<0?$ _____ $y>0?$ _____ answer ___

7. $8/5x+12 = 4/5x-15$ — $x<0?$ _____ $x>0?$ _____ answer ___

8. $\frac{x}{10} - \frac{2x}{7} = -12$ — $x>0?$ _____ $x<0?$ _____ answer ___

9. $\frac{3x}{8} - \frac{5x}{12} = 72$ — $x>0?$ _____ $x<0?$ _____ answer ___

10. $-5x+7x = -8x+\frac{1}{4}$ — $x<\frac{1}{4}?$ _____ $x>\frac{1}{4}?$ _____ answer ___

The estimation and solution of these problems lend themselves to group learning. They also show the effect of changes in parameters as stated in pg. 154 of the standards.

WHAT HAPPENS WHEN YOU CHANGE PARTS OF AN EQUATION?

1. $3x + 17 = 98$ Solve

   If the sum is 49 instead of 98, will the value of x be cut in half also? Guess.

   Try it and see.
   $3x + 17 = 49$

2. The sum of 5 times a number and 3 times a number is 48. Find the number.

   If the sum is doubled from 48 to 96, will the number be doubled?

   Try it and see.

3. a. The sum of 2 numbers is 78. If 3 times the smaller is added to the larger, the sum is 124.

   x = smaller 78 - x = larger

   What do you have to do to get answers just half as large?

   b. Will the value of the answers be cut in half if their sum, 78, is cut in half?

   Try it and see.

   c. Will the value of the numbers be cut in half if the sum, 124, is cut in half?

   d. Will the value of the numbers be cut in half if both 78 and 124 are cut in half?

1. your answer _____
   right answer 27

   yes ___ no ___

   new answer _____
   was the answer cut in half? ___

2. _____

   yes ___ no ___

   new answer _____
   was it doubled?
   _____

3. a. answer _____

   b. yes ___ no ___

   new answer _____
   was the answer cut in half? ___

   c. yes ___ no ___

   d. yes ___ no ___

SITUATIONAL PROBLEMS

Most problems in our mathematics books are designed as "exercises" for practice, and the students put just enough of their minds to work to solve them, then forget them.

The following problems provide situations that they might meet in everyday life. There is a bit of drama in each, enough to demand all of their attention.

1. To get a B grade, Pat must average 80 on the test score. up to now, the scores have been 70, 72, 82, and 85. Will a score of 92 on the final bring the grade to a B for the semester? What was his final average?

2. Your brother starts to walk to the bus station at 8:00. Ten minutes later, you realize that he has left his lunch and you start after him on a bicycle. If he walks at 5 km./hr., and you ride at 15 km./hr., will you catch him before he takes the 8:15 bus? How long will it take to catch him?

3. John's father is 26 years older than John. In 10 years, the sum of their ages will be 80. Is John old enough to get a driver's license in his state where the requirement is 18? Could he get one in your state? What was his age?

4. Fast Freddie already has one speeding ticket. His dad, the police, everyone agrees that one is his limit. Fred has been late for work two nights in a row. His boss says once more and he's fired. there are no stop signs between his house and work. The speed limit is 30 mph, and his house is six miles from his job. Will he keep his job if he leaves 10 minutes before he's due at work? If he leaves 15 minutes before? How long will it take him at 30 mph?

5. Your organization needs $2,500.00 for its operational cost during the next 3-month period. There are three groups in the organization. Because your group operates in a more favorable location in the neighborhood, you are to contribute 3 parts of the needed funds while the other two groups are responsible for 1 part each. Will your group reach its quota with $1,250.00?

   Yes _____ No _____

6. The number of towels taken by one of the occupants in a camp cottage was 6. This was one-tenth the total number of towels issued to the entire cottage. If the total number of occupants in the cottage is 55, would the number of towels this one occupant hoards cause a shortage among the others?

   Yes _____ No _____

   If yes, how many will be without a towel? _____

USABLE RATE FORMULAS

To find the average speed of a hot-rod on a $\frac{1}{4}$-mile drag strip, divide the elapsed time in seconds into 900. For instance, if it takes a car 10 seconds to go from a standing start to the quarter mile mark, it's average rate in miles per hour would be 900 divided by 10, or 90 miles per hour. Here are some problems:

1. What is the average rate in miles per hour if a hopped up Ford roared $\frac{1}{4}$ mile in 12 seconds?

2. A "Hemi under glass" made it in 9 seconds. What is his average speed?

3. What was the average speed in miles per hour, for elapsed times of (a) 8 secs.? (b) 9.3 secs? (c) 11.5 secs. ? Find answers correct to one decimal place.

The formula in problems 1,2, and 3 was:

$S=\frac{900}{t}$

S stands for average speed in mph

t stands for time in seconds for $\frac{1}{4}$ mile

Now if you want to find the time it takes to go $\frac{1}{4}$ mile at a speed of 80 miles per hour, you can change the formula around.

t=???

4. a. Solve the equation for t. b. Now find the number of seconds it takes to go $\frac{1}{4}$ mile if the car averages 60 mph.

5. What will the time for a $\frac{1}{4}$ mile be for the following speeds? (a) 45 mph (b) 90 mph (c) 48 mph (d) 110 mph.

(Note the extra motivation in this problem.)

6. The record for average speed on the $\frac{1}{4}$ mile for a track in Hot Rod City was 72.3 mph. Your friend's time was 12.2 secs. Did he break the record? What was his speed?

1. Your Answer

Correct Answer
75 mph

2. _____

3. Your Answer
a. _____
b. _____
c. _____

a. 112.5 mph
b. 96.9 mph

4. Your Answer
a. t = _____
b. _____

Correct Answer
a. $t=\frac{900}{S}$
b. 15 secs.

5. Your Answer
a. _____
b. _____
c. _____
d. _____

Correct Answer
a. 20 secs.
b. 10 secs.

6. Your Answers

Yes___No___

His speed?

_____

HIGHWAY SPEEDS

If the speed formula for hot rods over a quarter mile course is $s = \frac{900}{t}$, the formula for a full mile is $s = \frac{3600}{t}$, where s stands for the speed in miles per hour, t for the number of seconds. On many highways there are markers a mile apart.

7a. If the time for your car to travel between markers was 62 sec., what was its speed?

ans. 58 mph

b. If the time was 80 secs.? 1 min. and 15 secs.?

8. Sydney times how long it took between mileage markers and found that it took 53 seconds. Was the car traveling within the legal speed limit of 65 mph?

9. Another time it took one min. and 10 secs. between markers. The speedometer of the car read 55 mph. How accurate was it?

10a. What is the speed if it took just 60 secs. between markers?

b. How many seconds should it take between markers at 55 mph?
c. 65 mph? Make some guesses first.

Your guesses _____, _____.

11. If you travelled at 50 mph for 1 1/2 hrs., and 67 mph for 3 hrs., how far did you go altogether? _____

12. You are 90 miles away. How long will it take a friend to reach you at 60 mph? _____

13. You go for 2 hrs. and 15 min. at about 53 mph. Then your car breaks down. You call your dad to come and get you. How long will it take him at 63 mph? _____

14. You have gone 47 mph for 2 hrs. a. How long will it take a friend to reach you at 60 mph? _____ b. at 50 mph? _____

(Note the touch of drama in each of these problems. The answer to the calculation is necessary to finish the story.)

15. If you left home going 60 mph, and your brother left going in the opposite direction at 45 mph, how far apart are you in 2 hrs.? your guess _____ calculated answer _____ In 3 hrs? guess _____ answer _____

16. Over a long distance track, you have covered 327 miles in 3 hrs. If you keep going at that speed, will you break the record of 108 mph? yes ___ no ___

17. To finish a 600-mile race in 5 hrs., a. how far must a driver go in the first hour? _____ b. in 3 hrs.? _____

Other Applications

18. Five cassettes are on sale for 5 for $16. You have $3.25. If they are equally priced, do you have enough for one?

19. A fender assembly man must assemble 24 fenders in one 8-hour day. Will he make his quota if he assembles 2 in the first hour?

20. Fancy tigerskin upholstery is advertised at 6 yrds. for $80.00. How much will the material cost for each of the following jobs?

    a. A cushion, using one yard. _____

    b. The front seat, using 2 yards. _____

    c. The front and back seat, using 4 yards. _____

    d. For a complete job, including a cushion and paneling. this job takes 7 yards. _____

21. If your temperature is 38° Celsius do you have a temperature? (98.6 F is normal).

22. You are vacationing in January where the temperature is 29° C. Are you in Canada or Mexico?

23. Will your pond freeze at 8°C?

24. A 4" diameter ball of ice and juice sits on a 4" diameter cone 7" in depth. If you were forced to let the ice and juice melt would it overflow the cone?

25. Two ships leave port at the same time headed in opposite directions. The ship traveling north moves at 30 knots while the ship traveling south steams at 47 knots. Their radio equipment is sensitive enough so that they can remain in contact for 750 miles. Will they still be in radio contact 7 hours after departure? When will they lose contact?

26. The on-board computers of two passenger airlines flying at the same altitude have signaled the air traffic control tower that their flight paths can be described by the following linear equations:

    TWA: $2x + 5y = 20$ PSA: $8y = x + 16$

    Is there a possibility for a mid-air collision? If so at what point?

WHICH CAR HAS THE POWER?

PART I

Engine size is measured by the number of cubic inches or cubic centimeters of displacement. Horse power also depends upon displacement. Here is the formula. Use the table in Part II for questions in Part I.

Displacement = $.785\ B^2SN$

B= Bore or diameter of the cylinder

S= Length of stroke, the distance the piston moves up and down

N= Number of cylinders

1. Which has the greatest displacement, a Mustang 350 GT or a Barracuda S? Find the displacement of each car in cubic inches.

2. Which has the greatest displacement, a VW 1300 or a Volvo 122-S? Find the displacement of each car in cubic inches.

PART II

1. How many cylinders in a Honda S-600? It <u>is</u> an automobile.

2. What is the stroke in a Corvette 425? Find the answer to the neartest hundredth of an inch. Estimate your answers first.

Show on a separate sheet all your calculations for each problem.

	Bore Inches	Stroke Inches	Number of Cylinders	Displace. in Cu. In.	In Cubic c.m.
Mustang 350GT	4.00	2.87	8	_____	_____
Barracuda S	3.63	3.31	8	_____	_____
VW 1300	3.03	2.72	4	_____	_____
Volvo 122-S	3.31	3.15	4	_____	_____
Honda S-600	2.15	2.56	____	37	606
Corvette 425	4.02	____	8	396	6489
Mercedes 600	____	3.74	8	386	_____

3. What is the bore of a Mercedes 600?
One cubic inch contains 16.4 cubic centimeters.

For the Honda S, 37 cubic inches X 16.4 = 606 cubic centimeters

4. Now find the displacement in cubic centimeters for the Mustang, the Volvo and the Mercedes.

MEASURE AN EARTHQUAKE?

Measuring an earthquake may seem like an impossible job, but Dr. Charles Richter of Cal Tech has done it. It was hard because the range of amplitudes, the size of the motion, runs from 10 to more than 100,000,000 units.

You can make a Richter scale. Put a 10 in each blank space in the left hand column.

$(\ )^8$	
$(\ )^7$	
$(\ )^6$	
$(\ )^5$	
$(\ )^4$	
$(\ )^3$	1000
10^2	
$(\ )^1$	10

Now put the number associated with each power of ten in the right-hand column.

Imagine this: an earth quake of force 5 is 100 times greater than one of force 3. A quake of force 3 can barely be felt. But a quake of force 6 near a city will cause a great deal of damage. The Whittier, Calif. quake of October 1987 scored around 5.9, causing millions of dollars in damage.

Notice how from 10 to 100,000,000 the number are compressed into a small scale.

1. How many times greater is a force 5 than a force 4? _____

2. A force 7 than a force 4? _____

3. A force 8 than a force 2? _____

USEFUL POWERS OF TEN

Powers of 10 are all around you: \$100 is $\$10^2$; ten cents is 10^{-1} dollars. What is the highest power of ten that you can find around you? What is the lowest (most negative) you can find?

HOW MAY POWERS OF 10 DO YOU FIND AROUND YOU?

Consider \$1 as 10^0, 1 foot as 10^0, 100 feet as 10^2, etc. From this scale, answer the questions below.

10^{-3}	10^{-2}	10^{-1}	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7
.001	.01	.1	1	10	100	1,000	10,000	100,000	1,000,000	10,000,000

1. The smallest power of 10 I use _____. largest _____.

2. How many powers of 10 do your folks use? smallest _____

   largest _____

3. How many powers of 10 does a scientist use? smallest _____

   largest _____

4. What is the largest number you can make with 3 nines? _____

5. Which number is greater?

   a. $1^{2^{3^4}}$ or $4^{3^{2^1}}$ b. 2^{3^4} or 4^{3^2} _____

6. Is this operation associative?

   Does $(2^3)^4 = 2^{(3^4)}$? _____

7. Is the operation communative?

   Does $2^{(3^4)} = 2^{(4^3)} = 3^{(2^4)}$? _____

HOW FAR CAN YOU SEE?

1. From a tower 20 feet high, how far away is the horizon?

Your guess _____

With the distance, d, in miles, and the height of the tower, h, in feet, the formula is

$$d = \sqrt{\frac{3h}{2}}$$

How much is d if h is 20 feet? _____

2. If you double the height of the tower, will you be able to see twice as far? Guess: Yes____ No ___

Find d if h is 40 feet _____

Did the distance double? _____

3. Use the formula to fill in the table. Use the square root table if necessary. Graph it to understand better what happens.

Height in feet, h	0	2	4	6	8	10	16	20	30
Distance in miles d	0	1.17							

From your table estimate how high you have to be to see a distance of

Your Guess

4. 5 miles _____

5. 6 miles _____

6. 10 miles _____

Use the formula again to check your guesses to these questions. Solve for H, in terms of d, then substitute 4, 5, and 6 for d.

Ans. to 4._____ 5. _____ 6. _____

THE INSTRUMENT WAS A LITTLE OFF!!

1. Many measuring instruments are not exact. For instance, a certain thermometer always read 3.7° less than it should.

   For a reading of 56.2°, what is the true temperature? 56.2° plus 3.7° to correct the thermometer: 59.9°. If the actual temperature is 59° what will the thermometer read? 59° less 3.7° or 55.3°.

   a. If it reads 44.2°? _____

   b. -2.3°? (+1.4 ans.)

   c. -27°? _____

   d. Show the correct temperature for these readings:

Reading	57.2°	43.4°	0°	(-3.7°)	(-7.6°)
Actual Temperature	60.9°				

2. Another thermometer read 1.2° more than it should. Find the correct readings or temperatures for:

Reading	98.7°	52.1°	0°	-5.7°	-1
Actual Temperature	97.5°		1.5°	-3.5°	-1

3. John's long jump measured 21' 11". Then it was found that 10" had been cut off the tape! How far did he actually jump? The judge had to correct these readings:

   a. Measured 21' 11" 20' 9" 17' 4" 15' 0"

   Actual 21' 1"

   b. If he actually jumped 20' 10" what would the tape read? _____ c. If he jumped 20' 2"? _____
   24' 0"? _____

4. In changing from Standard Time to Daylight Savings Time, the clocks are moved up one hour in the Spring.

   Here is a formula: D = S + 1

Fill in the table:

Standard	8 AM	6 PM		7:30 PM	11:30 PM	
Daylight	9 AM		11 AM			1 AM

5. Weight watcher Jane's scales always read 2 pounds too much. --

Fill in this correction table for her.

Reading	110	112	113 1/2			122
Actual	108			110	115	

PROBLEMS OF A DISC JOCKEY

You are a disc Jockey for radio station XYZ.

1. How many tapes $3\frac{1}{2}$ minutes long can you play in $\frac{1}{2}$ hour?

2. How many could you play with a 30 second introduction for each? A 15 second introduction for each?

3. With a 15 second introduction and 10 minutes of commercials, how many could you play?

4. How much time could be added to the commercials to fillout the 1/2 hour?

5. How much time would the program have for commercials if the introduction of each of six records and its playing time was 3 and 3/4 minutes?

6. With 5 tapes, the time left for commercials was 8 minutes, what was the average time for each tape?

Answers 1) 8 with 2 minutes, over. 2) 7 with 2 minutes left over. 2b) 8 3) 5 and $1\frac{1}{4}$ minute, remainder. 4) $1\frac{1}{4}$ minutes 5) 7.5 minutes 6) 4.4 minutes or 4 miutes 24 seconds.

TURN THE AIR CONDITIONER DOWN

or

A MINUS TIMES A MINUS MAKES A PLUS

A heater is a plus agent, for it raises temperature. An air conditioner is a minus agent, for it lowers temperature. Right?!

An increase in power of either the heater or air conditioner is a plus; a decrease in power is a minus. Right?!

		Up (+)	Down (-)
Heater	(+)	+ times + = + It's Hotter	+ times - = - It's Cooler
Air Conditioner	(-)	- times + = - It's Cooler	- times - = + It's Hotter

Certainly, turning an air conditioner, a minus, down (also a minus) tends to increase the temperature, a plus result.

Also, a decrease, a minus operation, in unemployment (a minus situation), makes for a plus in the economy.

"Not ugly," "not bad," "not unlikely" are all double negatives which have positive meanings.

Examples such as these make "minus times a minus is a plus" a very sensible conclusion.

Which way, left or right, do you push the control to:

1. Make the room cooler? _____

2. Make it warmer? _____

3. Turn down the conditioner?
_____

4. a. Turn up the conditioner?
_____

b. Leave it for the night?
_____

Now practice on these:

5. $5(-3)$ 6. $3\cdot(5)$ 7. $-7(7)(-3)$ 8. $-5(-3)$

9. $-5(-3)(4)$ 10. $-7(7)(3)$ 11. $-8(x+1)$ 12. $-8(x-1)$

13. $-8(-x+1)$ 14. $-8(-x-1)$

answers:

6. 15 10. -147 13. $8x-8$

TO PROVE

$(a - b)(a + b) = a^2 - b^2$

An Investigation In Algebra

Purposes:

1. To firmly fix in the student's mind that the product of the sum and difference of two numbers is always equal to the difference of their squares.

2. To show how mathematical truth is established.

Method:

By trying number combinations first, verifying this truth through numbers, then going to symbols, the symbols will have their proper place in the student's mind.

Procedure:

Involves: $576^2 \lessgtr 575 \times 577$? Make a guess, we will come back to it later. Tell a story about the Arabs who might have been the first to ask such a question. This might have been the way they started out. (al jbar to algebra to Kareem Abdul Jabbar!)

Questions: The square, 8×8 is 64. How will that product change if we multiply $(8 + 1)$ and $(8 - 1)$?

Answer: The product is 63, one less than 8^2.

Will that difference of one increase or decrease as the numbers get bigger? Try 12^2 and $(12 + 1)$ and $(12 - 1)$.

Answer: No, the second product is still one less. Let's try a big number: 100^2 and $(100 + 1)$ and $(100 - 1)$. If a calculator is handy, check out $576^2 \lessgtr 575 \times 575$.

Answer: The product is still one less than the number squared.

Looks like we have a mathematical truth here. How about negative numbers, or zero? Try a few.

Answer: Still holds true.

Why does this have to be?

Answer: No matter what we put in for the number, the product will always be the number squared minus 1. The numbers in the middle always cancel one another. Let's use the symbol "a" to mean any number. Does it work for "a"?

$$\begin{array}{r} 9+1 \\ 9-1 \\ \hline -9-1 \\ 81 \quad 9 \\ \hline 81+0-1 \end{array}$$

Answer: For any number "a", the product of (a+1) (a-1) will always be the number squared minus one. It can't be anything else. The numbers that could change the answer always cancel out. But that is only the beginning. What if we add and subtract two? What will the difference be? Make a guess.

$$\begin{array}{r} a+1 \\ a-1 \\ \hline -a-1 \\ a^2+a \\ \hline a^2+0-1 \end{array}$$

Answer: The process is repeated. The difference is found to be four. How about three?

Let's call the number we add and subtract "b". Then $(a+b)(a-b) = (a^2-b^2)$.

Answer:

$$\begin{array}{r} a+b \\ a-b \\ \hline -ab-b^2 \\ a^2+ab \\ \hline a^2+0-b^2 \end{array}$$

Truth: For every pair of numbers, $(a+b)(a-b) = (a^2-b^2)$.

Special Cases: Try a = 0, b =3. Try a = 6, b = 9.

An Experiment with Pulleys

A simple pulley rig hung from the top of the blackboard with a number line, table of values, and chalkboard alongside it can be used to demonstrate every concept in linear transformations, $y = mx + b$. Functions, slope, intercept, and one-to-one correspondence come alive as the hand and pulley move up and down.

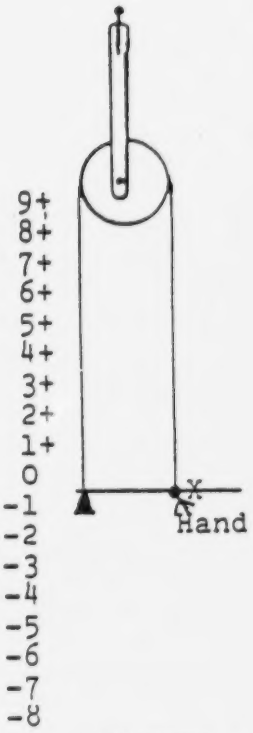


Figure 1

This rig also demonstrates what might happen in a factory or scientific laboratory as a physical situation is investigated. It is a visual, kinetic experience, putting the student right in the middle of a life-like situation.

The students are prepared with a ruler and graph paper with coordinate axes drawn and numbered. The teacher moves the hand up and down, the weight moves down and up.

Hand weight

x	y

Figure 2

Question: How do you describe this motion in English without mathematics? The students are asked to write some sort of answer. Most replies will be simple, like "If the hand goes up, the weight will do down, or vice versa."

Teacher: Now we will apply mathematics, make a mathematical model for this pulley, rig, "mathematize" it, so to speak.

To show us how much one changes with the other, we need an investigative team for experiment I.

Pat, you be the engineer, moving the hand and weight. John, you be the recorder with the table of values, and Helen, the analyst at the graph.

Pat starts with the hand and the weight both at zero. John puts zero, zero on the table of values, Helen marks the point on the graph. The class follows suit. Pat chooses another position for the hand, say +2, and reads the position of the weight, -2. The data is entered and another point is entered on the graph. Here is a good time to question the class for a prediction. Where will the weight go if the hand is put at say, +4?, -2? Pat moves the hand to check the estimates. The table of values is filled in, Helen marks the points and draws a connecting line.

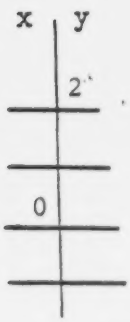
HERE ARE SOME QUESTIONS

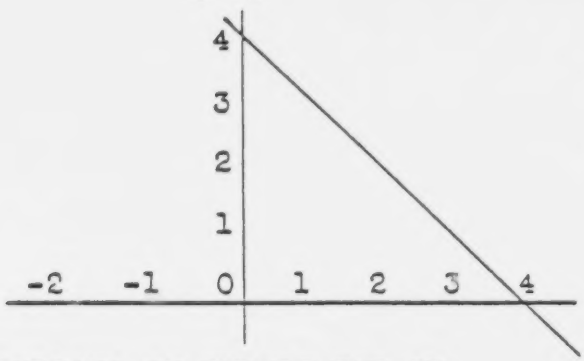
1. How is the position of the weight related to the position of the hand? _____
2. How can you express this relationship using x and y? y ___
3. What does the position of the weight depend on? _____
4. What does the value of y depend on? _____
5. If the weight is at some position such as -1, can x be anything except 1? _____
6. When related to the hand and weight, what does the minus sign in $y = (-1)x$ mean? _____
7. When related to the hand and weight, what does the 1 mean? _____
8. As x changes from 1 to 2, y, the position of the weight, changes from -1 to -2. What is the ratio of the change of the position of the weight to the change in the position of the hand? (Ans.: The weight changes -1 units, the hand changes 1. The ratio is -1.)
9. What is the value of the ratio in question 8 when the hand moves from 2 to 4, and the weight moves from -2 to -4? ____
10. If the hand is at some position such as -1, can the weight be anywhere except at +1? _____
11. If the weight is at some postion such as -3, can the hand be at any position except 3? _____
12. Is this a function? _____
13. If the hand is put at -6, where will the weight go? Make a prediction, then check, by moving the hand to -6. _____
14. If the hand is put at 5, where will the weight go? Predict, then check. _____
15. If the weight is put at 3, where will the hand go? Predict, then check. _____

EXPERIMENT II

"Now we will make a change in the pulley rig. For Experiment II, we will need another investigative team. George, you be the engineer, Idris the recorder, and Juan the analyst. This time, George, put the weight at plus 2, then put your hand on the cord at plus 2. Idris, fill in the new value while Juan puts the point on the graph. Now, George, move to a negative number for the hand and repeat the process. Try the hand at zero. How do the graphs compare? Where does the second graph cross the Y axis?"

George, put your hand at the zero position. Does the value for the weight check with the place where the graph crosses the Y axis? This is called the Y intercept, the value of y, when x is zero.





HERE ARE SOME QUESTIONS ON EXPERIMENT II

1. How are the position of the weight and the position of the hand related here? _____

2. How are x and y related here? Make a formula. y _____

3. Check your formula with the values in the table. Do they make your statement true? _____

4. The formula should read $y = -(1)x + 4$ or $y = -x + 4$. What does the minus mean? _____

5. What does the one mean? _____

6. What is the ratio of the change in y to the change in x? _____

This ratio is also the slope of the line on the graph. The y coordinate changes the same amount as the x coordinate, but in an opposite direction. One moves in a negative direction, the other in a positive direction.

7. In what manner, if any, has this ratio changed from Experiment I? _____ _____.

8. What does the ratio mean with respect to the pulley set-up?

_____

9. What is alike about the two graphs? _____

10. What is different about the two graphs? _____

11. What relation does the 4 in the equation $y = -x + 4$ have to the pulley set-up? _____

12. Where does this 4 appear on the graph? _____

13. What change would you make in the experiment to have the experiment come out a. $y = -x + 5$? b. $y = -x - 3$? c. $y = -x - 4$?

a. _____ b. _____

c. _____.

IMPORTANT POINT:

The coordinate, the number of the point where the line crosses the y-axis is called the y-intercept.

What is the y-intercept of the lines on your graph?

You can read the y-intercept from an equation in this manner:

$y = \frac{1}{2}x + (4)$ $\quad$ $y = -3x - (7)$

y-intercept $\quad$ y-intercept

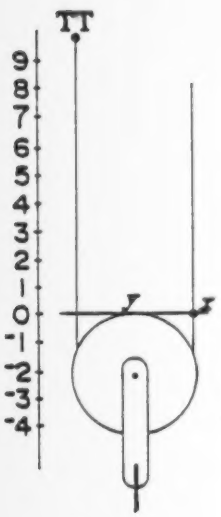


FIGURE 1

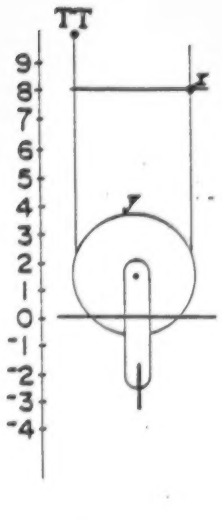


FIGURE 2

EXPERIMENT III

Set up the weight and pulley as in figure 1. Start the hand at zero and the weight at zero, using the top of the pulley as the point from which to take the reading. As before, make a table of values and a graph, using a new pair of coordinate axes.

The objectives of this experiment are to discover the changes in the equation resulting from a change in the rig.

QUESTIONS FOR EXPERIMENT III

1. How is the position of the weight related to the position of the hand?
2. How can you express this relationship using x and y?
3. Why is this a plus value when in previous experiments it has been a minus value?
4. When the hand, represented by x, is moved from 2 to 4, the weight moves from 1 to 2. What is the change in y, the position of the weight? What is the change in x, the position of the hand?

QUESTIONS FOR EXPERIMENT III - continued

5. What is the ratio of the change in the position of the weight to the change in the position of the hand? (Ans.: 1:2) (in the language of physics, this is the mechanical advantage of the pulley)
6. Where does this ratio appear on the graph? _____
7. What is the slope of the line on the graph? _____
8. Why does the graph slope up to the right in the graph instead of down as in previous graphs? _____
9. What is the smallest value of x that is possible here? (These pulley set-ups have limited domains) _____
10. Predict the position of the pulley if the hand is placed at 5. Check. _____
11. If the pulley was at $3\frac{1}{2}$, where would the hand have to be? _____

EXPERIMENT IV

With the pulley at zero and the hand at some other number, repeat the experiment, graph and answer the questions.

Why is an inclinded plane like a lazy dog?

An inclined plane in a slope-up.

REVERSE THE PROCESS

In these experiments, we started with a physical situation. To study it, we applied mathematics and developed an algebraic formula to describe carefully what happens when we change the position of the hand or weight, and what happens when we change the pulley rig. The question now is to start with a formula, then see if we can find a pulley rig to fit it.

1. How about $y=+x$? The sign in the first rig's equation is changed from negative to positive. How can we change the pulley rig so that the weight and hand go in the same direction? Or $y=+x-4$?

2. How can we change the second rig so that the equations are $y=-\frac{1}{2}x$, or $y=-\frac{1}{2}x+4$?

3. At what point will the number representing the hand be the same as the number representing the weight for a) $y=+\frac{1}{2}x$, b) $y=\frac{1}{2}x+4$, c) $y=-\frac{1}{2}x$, d) $y=-\frac{1}{2}x+4$.

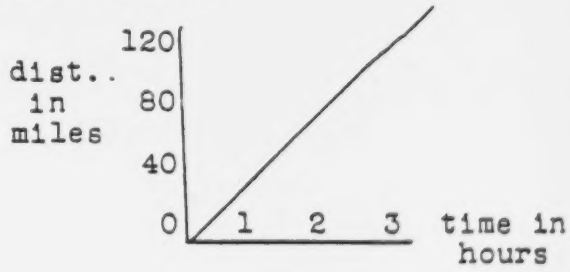
4. For a) $y=-x$, b) $y=-x-4$, c) $y=+x$, d) $y=+x-4$, state what the answer means for (c) and (d) in problem 4 for the graph and the pulley rig.

OTHER APPLICATIONS OF Y= MX+ B

In our algebra book we find many uses of y= mx+ b as a mathematical model, but they are not so indicated.

For instance, the special case of d= rt is a good example, when either r or t is constant. Take d= 40t.

t	0	1	2
d	0	40	80



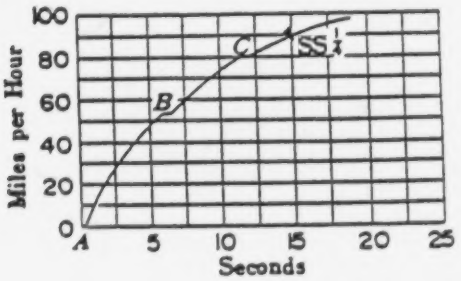
HERE ARE SOME QUESTIONS

1. How are the distance and the time related here? What formula shows this relation? _____ d= _____
2. What is the ratio of a change in distance to the corresponding change in time? _____
3. What is m in the general equation y = mx+b in this equation? _____
4. How does it show on the graph? _____
5. What is the value of b? _____
6. Consider the equation d = 40t + 8. What might the 8 mean here? _____
7. What is the slope of the line? _____

Consider the formula p = rB, where p is the percentage, r the rate, and B the base. Take the special case where r = .06.

1. How are the base and the percentage related here? _____
2. What is the ratio of a change in the percentage to the corresponding change in the base? _____
3. What is m, the slope in this equation? _____
4. What is b? _____
5. If there were a value for b, say $150, what would it mean? _____

WHAT IS THIS FUNCTION ALL ABOUT?



1. Under what circumstances are miles per hour compared to elapsed time in seconds? _____

2. What happens at the small horizontal segments B & C?

_____

3. Why is there a small horizontal mark "A" near the origin?

_____

4. Take a look at the graph between points B & C.
   a. Just what does the slope mean here? _____
   b. What does a change in y divided by the change in x mean? _____
   c. What is a change in miles per hour with respect to time? _____

ALGEBRA AND THIS BOOK

In calculating the cost, sale price, and possible profit for this book we used these figures:

Cost, each copy		One-time cost		Price	
Printing	3.00	Advertising	85.00	Income	6.50 each
Mailing each	.50				
Total each	3.50				

1. Where is the break-even point when the cost of production equals the income? Make a few guesses.

20 copies sold	Cost	Income
	20 x 3.50 + 85	20 x 6.50
	$70 + $85	
	Cost $155	Income $130

2. What would the profit or loss be for 50 copies? 50 x 3.50 + 85 ; 50 x 6.50
   $175 + $85
   a. What is the cost now? _____ b. What is the income? _____
   c. What is the profit or loss? _____

3. Trying many numbers is one way to find a break-even point. Algebra is another. Let n, some number, stand for the number of copies where we will break even, where cost will equal income

$$n \times 3.50 + \$85 = n(6.50)$$
$$3.50n = 6.50n - 3.50n$$
$$85 = 3.00n$$
$$n = 28.33 \quad \text{actual copies } 29$$

   If you sold 100 copies, what would the profit be?

4. What if we decide to add the cost of typing, $200, to the cost of the project? How do we change the equation?

   Answer: 3.50n + 285 = $6.50n (Typing is a one-time cost.)

   a. What is the break-even number now? _____
   b. For 100 copies, what is the profit this time? _____

5. We decide we need a proper cover, cost 6¢ each.
   a. Where do we add the 6¢? b. How does the cost equation change?

   c. What is the break-even point now?

6. You realize that you must lower your price to meet the competition, so the sale price becomes $6.25. 3.56n + 285 = 6.25n
   What is the break-even point now? What is the profit now on 100 copies?

7. Back to the situation in equation 4. At a convention, direct sale eliminates the mailing cost, which you decide to subtract from the actual cost and from the sale price. a. What is the equation now? b. What is the break-even point now? c. What is the profit on the sale of 100 copies?

USING MATHEMATICS TO INVESTIGATE THROWN BALLS, GRAVITY, AND FALLING WATER

LOUIS McCREERY

Throw a ball or other object straight up. Notice that it slows down, stops for an instant, then falls to earth with increasing speed. When you turn on a water faucet, note that the stream thins out as it falls. You know that a vase falling from a height of six feet is more likely to break than one falling from three feet.

The pull of gravity causes these phenomena, but how can we find out how high the ball will go, how long it will stay in the air, and how changes in the starting speed will effect the height of the throw?

We can find out by using a mathematical model, $h = rt - 16t^2$, for the flight of the ball. In our case, h is the height of the ball at some instant t in seconds, and r is the speed at which it starts its flight. The number 16 is a constant describing the pull of the earth on any object.

This formula is a member of the family of equations describing the force of gravity throughout the universe, affecting the sun, moon, and stars, as well as such unlike things as space shuttles and balls thrown at varying angles. Our formula is for the limited case of an object thrown straight up from the surface of the earth, ignoring air resistance.

1. To investigate mathematically how different starting speeds affect the height of the ball, start with 32 feet a second (about 22 miles an hour) for the value of r. The height

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at the end of each second or part thereof is given by $h = 32t - 16t^2$. In this problem, find the height for each half-second and depict the answers on a graph. Note that factoring will reduce the number of calculations from five to three. Use $h = 16t(2-t)$. Use two spaces for each second on the horizontal axis, and let a vertical space stand for four feet.

*

Time in seconds	0	0.5	1.0	1.5	2.0
Height in feet					

How long was the ball in the air? _____

How high did it go? _____

Answers for .5: $h = 32(0.5) - 16(0.5)^2 = 12$ feet

or $h = 16(0.5)(2-0.5) = 12$ feet

2. If you threw the ball up at twice the speed, would it go twice as high? _____ Four times as high? _____ $\sqrt{2}$ times as high? _____ Would it stay up twice as long? _____. To check your guesses, use the formula with the starting speed doubled: $h = 64t - 16t^2$ or $= 16t(4-t)$. Make a graph of these results.

Time	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height									

How many times higher did the second ball go than the ball in problem 1? _____

How long did it take to return to its starting point this time?

_____

*You may want to use copies of page 67 for the answers.

How close were your guesses? _____

3. Now double the starting speed again. Will the time the ball is in the air double? _____ Write a formula using 128 feet a second as the value for . Make a table. Your figures are approximate for the ball's travel in air, but they are accurate in a vacuum. At high speeds, the air resistance slows the ball appreciably.

Time	0	1	2	3	4	5	6	7	8
Height									

How high did the ball go this time? _____

How long was it in the air? _____

How close were your estimates? _____

4. Now that we have a pattern on the changes that can occur, use your tables and graphs from problems 1, 2, and 3 to answer these questions:

a. A ball is thrown upward at a speed of 16 feet a second. How long was it in the air? _____ How high did it go? _____

b. A ball is thrown upward at a speed of 8 feet a second. How long was it in the air? _____ How high did it go? _____

5. Now, let's think about falling water and falling objects. In problem 3, note the change in the distance traveled by the ball during successive seconds. How much of a change occurred from the end of second 4 to the end of second 5? _____ From second 5 to second 6? _____ The number of feet

the water falls a second increases and the water pulls away, as an accelerating auto pulls away from the rest of the traffic. In a like manner, the speed and, therefore, the destructive ability, of a falling object increases as it falls.

6. Did you know that you can tell how high a ball goes with a stopwatch? Suppose you threw a ball that stayed in the air for 3 seconds. From tables 1 and 2, make a guess as to how high it went. _____ At what speed did it start? _____ To find out, we can let $h = 0$, because the height will be zero when the ball hits the ground. From our graphs, we know that the ball is at a high point in one-half the total time.

$h = rt - 16t^2$

$0 = r(3) - 16(3^2)$

$0 - 3r - 144$

$r = 48$, so the starting speed is 48 feet a second

The formula for flight of the ball now reads $h = 48t - 16t^2$. At maximum height

$t = 3/2 = (1/2 \text{ of } 3)$;

$h = 48(3/2) - 16(9/4)$

$= 72 - 36$

$= 36$ feet at the highest point.

Now find the starting speed and the high point if the ball stays up 5 seconds--7 seconds.

a. For 5 seconds, the starting speed is _____ and the high point is _____.

b. For 7 seconds, the starting speed is _____ and the high point is _____.

In how many seconds will a ball thrown upward at 48 feet a second reach a height of 32 feet? You will get two answers. Why?

Solution: $h = rt - 16t^2$

$32 = 48t - 16t^2$

$16t^2 - 48t + 32 = 0$

$16(t^2 - 3t + 2) = 0$

$16(t - 2)\ (t - 1) = 0$

$t = 2;\ t = 1$

The following are more problems for investigation.

7a. In how many seconds will a ball thrown upward at 64 feet a second reach a height of 48 feet?

Your Guesses _____ Answers _____

b. In how many seconds will a ball thrown upward at 50 feet a second reach a height of 25 feet?

Your Guesses _____ Answers _____

8. On the moon, the pull of gravity is one-sixth that of the pull of earth, so a thrown ball will behave quite differently there. For a starting velocity of 32 feet a second, as in problem 1, guess how high the ball would go on the moon. _____

Using the formula $h = 32t - 1/6(16)t^2$, or $h = t(32 - 2.67t)$, fill out the table.

t	0	1	2	3	4	5	6	7	8	9	10	11	12
h													

How close was your guess? _____

To coin a word, we have "mathematized" the flight of such objects as a ball and falling water under the influence of gravity of the earth and of the moon, with different starting speeds. We have discovered a means of finding the height reached

by a ball thrown upward given its time in the air. Without mathematics, such an investigation would be impossible.

Mathematics is beautiful in itself and a most useful tool.

MATHEMATICS IS ABOUT LIFE

It was a great day when our astronauts landed on the moon. It was a great day for mathematics, for without it the mission could not have been conceived or carried out. Mathematics is one of the main bases of our life and our progress. Without it, we would still be in the stone age, bartering for goods.

With it, we sent a probe to Uranus and beyond; without it we would still be hunting with bows and arrows. Without mathematics, no high finance or football scores. Anyone asking "how much" or "how many" would be answered with guesses or such words as "heap."

Open your eyes to how great it is.

INVESTIGATION PLAN FOR THROWN BALLS AND GRAVITY

General Formula: $h=rt-16t^2$ or $h=t(r-16t)$ h in feet; r in ft/sec; t in sec

1. $h = 32t - 16t^2$ or $h = 16t(2-t)$ Fill in table and graph

Time in seconds	0	0.5	1.0	1.5	2.0
Height in feet					

r = 32, total t = _____
greatest height = _____

2. How much higher will it go if you double the starting speed to 64 ft/sec?
How long will it stay in the air?

Guesses: time ____ height _____
r = 64, total t _____
height _____
How good were your guesses? _____

$h = 64t - 16t^2$ or $h = 16t(4-t)$

Time	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height									

3. Double the starting speed again.
How many times higher will it go than in problem #2? We ignore air resistance.

Guesses: time ____ height _____
t = 128, t = _____
height _____
How good were your guesses? _____

$h = 128t - 16t^2$ or $h = 16t(8-t)$

Time	0	1	2	3	4	5	6	7	8
Height									

4. Here we divide the starting rate in problem 1 by 2. From that table you can find t and h for r=16 and 8.

a) r=16, time____ h=_____
b) r=8, time____ h=_____

5. Note acceleration in problem 3. Second 4 to 5 _____ Second 5 to 6 _____

6. To find out how high a ball goes if it stays in the air 5 seconds set h = 0, and t = 5. To find r, set up an equation, find the highest point. Repeat for 7 seconds.

If t=5 secs., r=maximum h= _____
t=7 secs., r=maximum h= _____

7a. At what times will a ball thrown upward at r = 64 ft/sec be 48 ft high? Find for r = 50 ft/sec, h = 25 ft.

a) r=64, t=__, __, h = 48
b) r=50, t=__, __, h = 25

8. If the pull of gravity on the moon is sixth of the pull of gravity on the earth, how many times higher will a ball thrown upward at 32 ft/sec go on the moon than on the earth? Your guess _____

Actual

$h = 32t - \frac{1}{6}(16)t^2$ or $h = t(32-2.67t)$

Time	0	3	6	9	12	15
Height						

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Louis McCreery

APPENDIX

HOW CASEY GOT HIS BATTING AVERAGE

BY LOUIS R. McCREERY Los Angeles Times

One facet of baseball which continues to amaze me is the mass of statistics on teams and players that has been collected over the years and which are continually quoted by sports writers, broadcasters and quiz contestants.

Something so complete and complicated took years to develop, and must have had its start in the beginnings of the game. It might have started this way:

In the early days of baseball, when there was still joy in Mudville, the supporters of Casey made the best argument for their man, sometimes using truth, sometimes depending on the volume of their cheering and their noise. Close behind them were the Benson Bend Batsmen fans of their star O'Leary. These worthies insisted that their man surpassed them all.

★

After one game, when the Mudville Marauders had dropped a close one, thanks to a grand slam by O'Leary, Casey's fans increased the volume, if not the logic, of their praise, perhaps to bolster their total faith in him. Amid the clamor sat a devoted fan, Joe, who was also a mathematician, and in sharp contrast to the rest, this fan was wrapped in thought.

"There should be some way to prove that Casey is the better ballplayer," Joe mused, "some system of numbers, lengths, ratios or percentages, or perhaps a combination of such. We ought to be able to really show 'em with something like that."

At that moment here occurred to him a theory which has been immortalized in acres of figurs in sports pages throughout the years. He stirred himself and shouted, "Eureka! I know how to prove who is the better man!"

Joe's friends didn't share his enthusiasm. "We know who the better man is," they sneered. "It's the Benson Boys and the rest of the league that won't be convinced. Back to your numbers."

Our mathematician decided on a less scientific approach.

"Gentlemen," he pleaded, stretching the definition slightly, "gentlemen, I know how to prove Casey is the better man."

The chatter diminished somewhat. "How?" said one tanned veteran, still skeptical, but indicating a willingness to try for more solid proof.

"We could count the number of hits, homers and the like."

"They tried that once before and made O'Leary look better because he played more games. You can't prove a thing like this with numbers." The veteran turned away, and the noise began to increase again. But before it engulfed him, Joe managed to yell, "How about ratio?"

That slowed the noise again. "Ratio? What's ratio?"

Joe pressed his advantage. "Ratio, you know, like number of hits for 25 times at bat, or percentage of hits for total times at bat. That will eliminate the difference in total games."

He watched to see if the idea took hold. The noise level dropped to a new low, and here and there he watched an eye squint and a thoughtful hand twirl a mustache. This was a serious matter, due full consideration.

Soon came a measured response. "That might be a good idea. We ought to try it for all the good hitters in the league. Johnson of Gopher Gulch really belts them out, too. He might even beat O'Leary." With that pleasant prospect the group was sold.

The plan sounded good to the league president, too, who put Joe in charge of records. The league offered a prize to the hitter with the highest batting percentage for the coming season, which looked to many to be the most exciting development yet. Joe was pleased but wary, for being a good mathematician he knew that mere mortals can't foresee all in the scheme of things, and there might be some untoward developments. It just might be that the system would show that Casey was NOT on top! The thought, like a cold autumn wind, made him shiver, but there was no turning back.

The next season was indeed exciting, and the batting lead bounced around among several men. Joe kept his slide rule busy. As the season neared its close, sure enough, Casey and O'Leary were well out in front, tied for first.

Then it happened. The score was tied, last half of the ninth, one man on, Casey up. He needed this hit to finish ahead of O'Leary. But he was walked, deliberately walked. An intentional walk is a pitcher's compliment to a hitter's power and is good baseball. But this was a dirty, dirty trick. You can't get credit for a hit if they won't let you hit. Casey broke his bat on the nearest post.

★

O'Leary was soon in a rage, too, for when he came up, his manager called for a sacrifice. O'Leary, thinking of that average, chose to swing with all his power at a high one, missed, and was fined $20. To top it all came word that the batting championship had been won by a newcomer who had scratched out a single in his only time at bat for the season. His percentage was 1.000!

This bit of news sent Casey's loyal fans heading with blood in their eyes for Joe. He managed to hold the baseball bats at bay with his slide rule until tempers cooled. He then suggested that perhaps walks and sacrifices should not be counted as times at bat, and that a batter should be required to bat at least 500 times to qualify for a batting crown.

With these changes, enough objective thinkers were satisfied that it was worth another try. Now the system is an honored part of baseball, and incidentally a good mathematical model.

Such could have been the beginning of it all.

BEST COPY AVAILABLE

I'M THINKING OF A NUMBER----X

When introducing a new concept, good teaching will find a connection to an idea or process with which the student is already familiar, not only in mathematics but in any phase of the student's life. When introducing variables and equations, number riddles are such a connection.

The questions in the following scenario are followed by descriptions of what possibly happens in the student's mind. Some of the ideas come from Blais' article in the November 1988 issue of the Mathematics Teacher, "Constructivism - a Theoretical Revolution for Algebra" and Dr. Glenda Lappan's presentation at the November 1988 conference of the Southern Section of the California Mathematics Council. Terms in quotes are from Blais' article.

Teacher: I'm thinking of a number.

The "essence" of variable or unknown is contained here right in the student's experience. No further explanation is necessary. Connection is made to a thought pattern already existing.

Teacher: ... add 5 to it and you get 12. What is my number?

Most will answer with 7.

Teacher: What did you do to get that answer?

Answers will vary. The students are asked to investigate their own thinking patterns; the teacher can become aware of them, "Anybody knows 7 + 5 = 12," "I subtracted 5 from 12."

Teacher: Twice my number is 24 ... I add 9 to mine and I get 20.

A few of these with explanation of thinking patterns and the class is asked to offer some of their own. To give everyone a chance at solving and creating, the class is broken up into groups. The "essence" of equations, both of solving and creating, is experienced and made the student's own. They become "experts". There is no mystery here; symbols and new words are not yet introduced.

Teacher: How about 2 times my number plus 1 is 25?

This takes a little longer.

Teacher: What did you do here?

Some will divide by 2, then adjust the answer to fit. Others will subtract the 1 then divide. The better ways become apparent; some will accept a new patern. The class is lead to more complicated riddles, finally bogging down. Some might try writing something on paper.

Teacher: Would it help to use a pencil and paper? Try this one: (or one presented by the class) If three-quarters of my number and 16 are added the result is the sum of 11 and 14. What is my number? Let's write that down.

Teacher demonstrates, perhaps with students writing along with him or her as he or she writes and narrates.

Three-quarters of my number plus 16 is the sum of 11 and 14:

$$\frac{3}{4}(?) + 16 = 11 + 14$$

What has happened here? Without mention of new words, concepts or symbols, the class has experienced algebra, finding the concept of variable and equation in their own minds and are now ready for more difficult work. They know what they are doing; they are aware of their own thought processes.

"Knowledge is something which each individual must construct for and by himself. This view of knowledge as an individual construction ... is constructivism."

From a study of very recent literature, I find that the work is not only good for motivation but illustrates some of the present day philosophy. We are moving away from "modern math" toward a more student-centered approach.

For instance, Dr. Harold C. Schoen of the University of Iowa, in his chapter on "Teaching Algebra with a Word Problem Focus" in the 1988 NCTM Yearbook "The Ideas of Algebra", listed these recommendations for teaching principles:

No. 1 - "Build new learning on students' existing knowledge and understanding."

The thrust of Motivators is to build not only on knowledge but on the whole person (see introduction) - "start where the students are."

No. 2 - "Lead gradually from verbalization to algebraic symbolism."

Motivators pp. 71 1, 2, 49 fit his principle exactly, coming closer to where "the student is" than any material I have seen.

No. 3 - "Introduce algebraic topics with applications."

The pulley investigation p. 51ff is a good example. It is visual and "hands on," providing a laboratory experience, taking the students from the raw material to the finished product. It was the subject of an

article in the Mathematics Teacher and a TV program and is the most complete treatment of a single subject I have ever seen. It was most useful during my years at San Marino High School.

No. 4 -"Teach algebraic topics from the perspective on how they can be applied."

· See pp. 37-40 , "punch line" problems, where the solution is put to immediate use. This is an original way of constructing problems, as far as I know. It is a strong motivator.

The Mathematics Teacher is a good indication of the trends in Teaching philosophy. The October 1988 issue's lead article was on investigation. Page 49 (a^2-b^2), again pulleys, p. 51 ff, and gravity, p. 62 ff are investigations.

The November 1988 issue features "Constructivism - A Theoretical Revolution for Algebra." To quote, "Constructivism ... says that knowledge is something that learners must construct for and by themselves ... Discovery, reinvention, or active reconstruction is necessary."

We add on p. i questioning, conjecture, investigation, and estimation. "Don't tell them, as them." We seek personal as well as academic reasons for the student to be interested in a problem. And to quote a student, "Don't tell me anything I can find out for myself."

"Situational lessons" are another current emphasis. As you may notice from the title they are a basis of this work.

4cm x 14cm

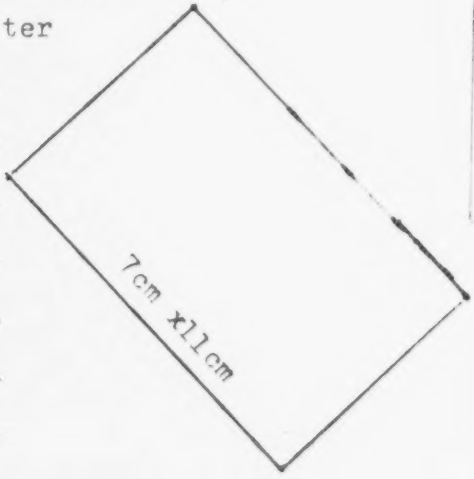
6cm x 12cm

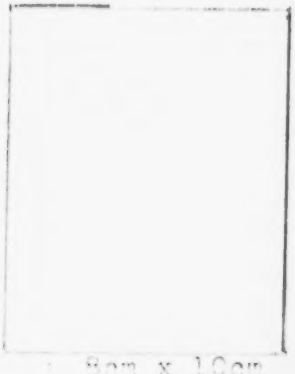
9cm x 9cm

WHICH IS THE LARGEST RECTANGLE ?

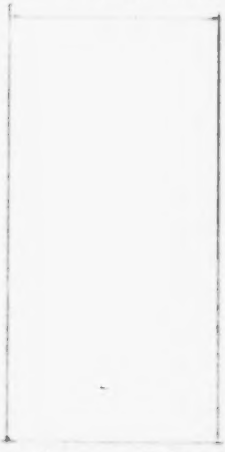
Make a guess. Which of these rectangles would take the most paint to cover it? Which has the largest area? Find the area and perimeter of each. (Some are not shown)

Length	Width	Area	Perimeter
4 cms	14 cms	____	____
5	13	____	____
6	12	____	____
7	11	____	____
8	10	____	____
9	9	____	____
10	8	____	____
11	7	____	____





8cm x 10cm



6cm x 12cm

4cm x 14cm

1. Using your answers these problems, make a guess about the area of a rectangle 7.2 x 10.8 cms.____

   Find the exact answer_____.

2. Estimate, then find the areas of

   a. 7.8 x 10.2 inches. Est._____ans._____

   b. $6\frac{1}{4}$ ft. x $11\frac{3}{4}$ ft. _____ _____

   c. 8.5 x 12.5 inches _____ _____

3. Bonus Problem: The circumference of circle is also its perimeter. If the perimeter of a circle is 36 cms, as in the problems, will its area be greater or less than that of the largest rectangle? To find the area of the circle, divide the square of the circumference, 36, by 4pi.

(see page 49 for theory)

Ans. Areas: 56,65,72,77,80,81,80,77 sq.cm Per. all 36cm.
1. 77.76 sq. ... 279.56 sq. in b. 73 7/16 sq.ft.
c. 68.75 sq.in. 3. 103.2 sq.cm.

ANSWERS

MOTIVATORS FOR ALGEBRA ONE

ANSWERS

p. 1 1) 3 2) 11 3) 11 4) 7 5) 30
6) 7 7) 14 8) 8 9) 36 10) ?

p. 5 Multiply or divide by a negative number.

p. 7 1) 45 2) 75 3) 63 4) 2 5) 64
6) 10.5 7) 100 8) 13 1/3 9) 15 10) 137.5
11) 15.12 12) 37.5 13) 14 14) 21 15) --

p. 8 1) 30 2) 30 3) 0 4) 11 5) 11
6) 12 7) 10 8) 20 9) 1

p. 9 2,4,6,8

p. 10 1) yes 2) no 3) no 4) no 5) no
6) no

p. 11 1) 1,4,8,10 with Common Factors 1,4,10

p. 12 1) $x + 1$ 2) 4/3 3) $x + y$ 4) no change
5) no change 6) x 7) no change 8) $2x - 1$
9) $\frac{4x^2 - 1}{(x - 1)}$ 10) $17(a + 2)$ 11) $\frac{a + b}{a(a - b)}$

p. 13 1,2,5,7,13

p. 14 1,3,5,6,8,10,12,14,17,22,24

p.15 1,2,4,5,6,8,9,10,12,13,15,16,17,19,20,23,24,25,26

p.16 28,31,33,36,41,44

p.17 Part I: circled: 2,4,6,8,9,11,13

Part II: not indicated addition: 3,8,12

1) a 5) 7b 7) -4b 13) 0

p.18

1) +1	9	9	9	2) yes
-1	1	1	1	3) different form of same expression
0	4	4	4	
2	16	16	16	4) they are perfect squares
-2	0	0	0	5) different form of a square
5	49	49	49	6) left 6a) 26 6b) 4
$-\frac{5}{2}$	1/4	1/4	1/4	p.19 1) yes; 2b) false; 5,7) false

p.21,22

1a) $15,000

1b) $ 630

2a) 1600, 1800

2b) 70, 560

3a) 450 mph, 50 miles
900 miles, 950 miles

3b) s, 2s, 10s, 30s, 630 miles
2s + 30s = 992, 31mph,
310 mph

4a) 90, 306, +42

4b) 361

4c) 8, -9; 4, -5; 11, -12

5a) 105om, 1925m, 1623m

5b) 3 hrs., 3.5 hrs.

5c) 428.6 mph

6a) 2325m, 3487.5

6b) 4 hrs.

6c) 292 mph

7a) 8 yds, 24 yds, 32 sq.yds.

7b) 6yds, 12yds, 72 sp. yds.

8)

w	l	p	A
(6)	13	38	78
(10)	21	62	210
7	(15)	44	105
11.5	(24)	69	276
8	17	(50)	136
1.5	4	11	(6)
2.5	6	17	(15)
3.5	8	23	(28)

(Given facts circled)

p.23 b) $4/9\sqrt{6}$ d) $a/4\sqrt{2x}$ e) $1/2\sqrt{30}$

p.26 1) $8, 9, ax^4, x^3, 11, ac, 45$

2) $3\sqrt{3}$, $4\sqrt{2}$, $10\sqrt{2}$, $6\sqrt{2}$, $x\sqrt{x}$, $a\sqrt{x}$

3) $1/4$, $1/5$, $1/x$, $\frac{1}{x^2}$, $\frac{1}{a}$

4) $\frac{\sqrt{3}}{3}$, $\frac{\sqrt{3}}{9}$, $\frac{\sqrt{x}}{x}$, $\frac{1}{a^2}\sqrt{a}$, $\frac{\sqrt{30}}{5}$, $\frac{\sqrt{3}}{3}$, $\frac{\sqrt{7}}{7}$

p.28 1) 30 sq. ft. 2) 60 sq. ft. 3) 120 sq. ft. (4 times as large)

4a) 270 sq. ft. 4b) n^2 4c) 480 sq. ft.

4d) 750 sq. ft.

p.29 5) 6.555 sq. cm. 6) 21 51/64 or 21.8 sq. in.

7) 31.72 sq. in., 1.72 sq. in.

p.30

	area sq. in.	volume cu. in.	
1a)	6	1	1d) Area: 4 times as large or 2^2
1b)	15.36	4.096	
1c)	24	8	Volume: 8 times as large or 2^3

2a)	61.44	32.768	2e) 3.2 is 2 x 1.6, area increases by 2^2
2b)	216	216	2^2 x 15.36 = 61.44
2c)	600	1000	volume increases by 2^3, or 8 x 4.096 = 32.768
2d)	864	1728	

p.31 1a) 84 cu. in 1b) 168 cu. in 1c) 336 cu. in

1d) 672 cu. in 1e) 91.728 cu. in, inc. 7.728 cu. in

p.32

	circum. inches	area sp. in.	
1a)	44	154	1c) Area increases faster
1b)	88	616	
1d)	32.656	84.8156	

2a) 113 1/7 cu. ft. 2d) 2^3 or 8

2c) 906 1/7 cu. ft.

3a) 33.5 3b) 38.7

3c) increase: 11 cu. in. 3d) 3^3 or 27 times

3e) n^3

p.33 Fancy Formulas

1a) 28 1b) 28 1c) 8,27

2a) 140 2b) 285

3a) 135° 3b) 140°, 5°

p.34 3c) 144° 3d) approaches 180°

p.35 1) 6 3/7 2) -1 3) 9 4) 29 1/3 5) 4 4/15

6) 4 1/14 7) -33 3/4 8) 64.6 9) -1728 10) 1/40

p.36 1) 10 2/3 no 2) 6,12 yes 3a) 23,55

3b) no 42 1/2 3 1/2 3c) no -8,82

3d) yes 11 1/2, 27 1/2

p.37 1) yes, 80.2 2) yes, in 15 min. 3) age 17 4) no, yes

5) no, $250 6) no, just enough

p.38 Usable Rate Formulas

2) 100 mph 3c) 78.3 mph 5b) 10 secs.

5c) 18.8 secs 6) yes, 73.8 mph

p.39 Highway Speeds

7b) 45 mph, 48 mph

8) No, 67.9 mph

9) Actual speed 51.4 mph
speedometer 3.6 miles fast

10a) 60 mph, mile a minute

10b) 65.4 secs. 10c) 55.4 secs.

11) 276 12) 1 ½ hrs. 13) 1.9 hrs

14a) 1.57 hrs. 14b) 1.9hrs.

15) Yes, 210 miles, 315 miles 16) yes, 109mph

p.40 17a) 120 mph 17b) 360 mi 18) $3.20 each, yes

19) no 20a) $13.33 b) 26.66 c) $53.33 d) $93.31

21) yes, 100.4° F 22) Mexico, 84.2 F 23) No, 8° C is 46.4 F 24) Yes, Ball 33.4 Cu. In. Cone 29.2 cu. in.

25) Yes, 9 hrs 45 min. after departure 26) Yes, (3.8, 2.95)

p.41 PART II

	cu. in.	cu. cm.	
a)	298.4	4614.0	e) 4 cyl.
b)	273.9	4492.1	f) stroke 3.77 in.
c)	78.0	1286.0	g) bore 4.85 cm. 6330.4 cc.
d)	108.4	1777.2	

p.42 Reading up 10, 100; 1,000; 10,000; 100,000;

1,000,000; 10,000,000; 100,000,000

1) ten 2) thousand 3) million

p.43 1-3) answers vary 4) 9^{9^9} 5) $4^{3^{2^1}}$

5b) equal 6) no, $2^{12} \neq 2^{81}$ 7) no, $3^{81} \neq 2^{64} \neq 3^8$

p.44 1) 5.47 mi. 2) 7.75 mi. No

3) Ht. 4 6 8 10 16 20 40

Dist. 2.45, 3, 3.46, 3.9, 4.9, 5.5, 6.7

4) 16.7 5) 24 ft. 6) 67 ft.

p.45 1a) 40.5°

c) -23.3°

d) 47.1°

3.7°

0

- 3.9°

2) 50.9°

- 1.2°

2.7°

-6.9°

-2.3°

-2.2°

+ .2°

3a) 19' 11"

16' 6"

14' 2"

b) 21' 8"

c) 21' 0"

d) 24' 10"

p.46 4) 7:00PM

10:00AM

8:30PM

12:30AM

12:00MIDNIGHT

5) 110

111 1/2

112

117

120

p.48 Turn the Air Conditioner Down

1) left 2) right 3) right 4a) left b) left

5) -15 7) +147 8) +15 9) 60

11) -8x-8 12) -8x+8 14) +8x+8

p.51 Possible Values

x	y
0	0
+2	-2
-3	+3
+7	-7

p.53 1) The number representing the position of the weight is always the opposite or negative of the number representing the position of the hand.

2) y = -x 3) the position of the hand

4) the value of x

5) no 6) see answer to question 1

7) The weight will move one unit when the hand moves one unit.

8) answered

9) -2/2 10) no 11) no 12) yes

13) +6 14) -5 15) -3

pp. 54, 55

1) The sum of the numbers representing the hand and weight is always 4.

2) y = -x + 4 3) yes

4) The hand and weight move in opposite directions

5) Each will move the same number of spaces, or the numbers will change the same amount.

6) -1 7) no change

8) The weight and hand move the same amount in opposite directions. 9) same slope

10) Different intercepts when the pulley is at zero. 11) The weight is at 4 12) on the Y axis

13) Start with the hand at zero and the weight at the new intercept.

p.56 1) The weight changes position one-half as fast as the hand.

2) $y = 1/2x$ 3) the hand and weight go in same direction.

4) the position of the weight changes one unit when the hand changes two units.

5) answered 6) in the slope of the line

7) 1/2 8) slope is positive. The hand and weight move in the same direction.

9) 0, perhaps - 1/2 10) 2 1/2 11) 7

p.57 Experiment IV. Starting with the hand at zero, and the weight at -4.

p.58 In exp. 1, loop the cord under a finger. The hand and weight will move in the same direction. Or put the hand directly above the weight, eliminating the pulley.

2) loop the cord over a finger in Exp. III

3) all points b.(8.8) c) (0,0) d. $(\frac{8}{3}, \frac{8}{3})$

4) a) (0,0) b. (-2,-2) c) all points d. no points No rig possible

1) The number representing the position of the weight is always one-half of the number representing the position of the hand, with four subtracted from it.

2) $y = x - 4$ 3 thru 8) same as Exp. 2

9) -8 10) 11

p.59 To find the distance in miles you multiply the time by 40. d=40t

2) 40 3) 40 4) the line moves up 40 units for every hour of change. 5) zero

6) distance away when the time started 7) 40mph

1. The percentage is the product of the rate times the base.

2. 6/100 3. .06 4. zer 5. Money owned but not drawing interest

p.60 1) hot rod race 2) racer shifts gears

3) wheels spin at the start 4a) acceleration

4b) The amount the speed of the car is increasing per second.

4c) Acceleration

p.61 1) loss $25 2a) $325 2b) $260 2c) profit $65

3) $215 4a) 95 4b) $15 5a) to the cost of each copy

5b) first term becomes 3.56n 5c) 97

6a) 106 6b) loss of $47

7a) 3.00n + 285 = 6.00n 7b) 95 7c) $15 no change from prob. 4

p.63 1. 0, 12, 16, 12, 0; 2 sec.

2. 0, 28, 48, 60, 64, 60, 48, 28, 0; 4 times, 4sec.

p.64 3. 0, 112, 192, 240, 256, 240, 192, 112, 0; 256 ft. 8 sec.

4a. 1 sec.; 4ft.; 4b. 1/2 sec.; 1 ft.

5. 16 ft., 48 ft.

p.65 6. 80 ft./sec., 100 ft.; 112 ft./sec. 196 ft.

p.66 7a. 1 and 3 sec.; 7b. 5/8 and 5/2 sec.

Moon

t	0*	1	2	3*	4	5	6*	7	8	9*	10	11	12*
h	0	29.33	53.32	72	85.28	93.25	96	93.25	85.28	72	53.32	29.33	0

* Answers for investigation sheet p.68